

Complementarity (substitutability) of Goods: in the search of a positive government spending multiplier*

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Abstract

By quantifying the extent to which economic growth is affected by changes in government spending, the government spending multiplier (GSM) derived in a given context emerges as one of the potential barometers of the effectiveness of fiscal policy, particularly in developing and emerging economies. This paper evaluates the magnitude and sign of the government spending multiplier for the Caribbean countries over the period 1993Q1-2019Q4. The GSM is derived after fiscal expansion in the context of a dynamic stochastic general equilibrium (DSGE) model. The paper contributes to the debate on the size and sign of the GSM by showing, *inter alia*, that the latter depends on country characteristics such as the degree of complementarity (substitutability) between private consumption and public consumption or expenditure, and the elasticity of labour supply. The empirical results have policy implications.

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Key Words: Fiscal Multiplier, Fiscal Policy, DSGE, Government Spending Shocks.

*This paper presents preliminary findings and is being distributed to economists and other institutions solely to stimulate discussion and comments

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1 Introduction

Fiscal policy is one of the policies used to influence the pace of an economy in a meaningful way. Its impact is usually measured by the so-called fiscal multiplier (government expenditure and/or tax multiplier), a concept that is both simple in definition and complex in operationalisation. In fact, a fiscal multiplier is simply the response of output to a shock or a change in an exogenous fiscal instrument (government expenditure or tax). While the definition is quite clear, its operationalisation is challenging because the fiscal multiplier is not really a structural parameter, but rather a hybrid parameter in the sense that it is a function of structural and policy response parameters (Chinn (2013)). There are, of course, several schools of thought that have tried to shed light on the phenomenon of the *fiscal multiplier*. There are the Keynesians, the neoclassicals, the new classicals and the new Keynesians, to name but a few. In the same way, several methodologies have helped to calculate fiscal multipliers. Three approaches are important and, at least for the moment, somewhat less important. These are the simultaneous equations model (SEM) approach *à la* Cowles Commission, the vector autoregression (VAR) family approach, including structural VAR (SVAR), the model-based approach, mainly represented by dynamic stochastic general equilibrium (DSGE), the narrative approach and the bucket approach.

Interest in the fiscal multiplier was revived in the aftermath of the 2008 global financial crisis and in response to the coronavirus pandemic, which left a number of countries in dire economic straits. This implies that measures need to be taken to revive these economies. In this context, the size and sign of the fiscal multiplier is of ultimate importance, as it can sensibly influence the advice and design of policies, as well as the accuracy of macroeconomic forecasts (see Batini, Eyraud, and Weber (2014)). In any case, determining the size and sign of fiscal multipliers has been the subject of a considerable number of studies. It is widely accepted that an increase in public consumption has an impact on output. The sign and direction are still the subject of theoretical and empirical debate. The main issue is whether public and private consumption are complementary goods (Edgeworth complementarity) and the implications for the sign and size of the fiscal multiplier. If this is the case, an increase in public spending increases the marginal utility of private consumption, *ceteris paribus*. This could lead to an increase in labour supply. On the other hand, Edgeworth substitutability between the two goods is a sufficient condition for the increase in public consumption to have a crowding out effect on private consumption. Some studies have focused on the degree of complementarity between private and public consumption. For example, Coenen, Straub, and Trabandt (2012) and Feve, Matheron, and Sahuc (2013) introduce a utility function in which these two variables are non-separable and find a strong degree of complementarity between them.

Others include Guajardo, Leigh, and Pescatori (2014), Cloyne (2013), Mertens and Ravn (2013), Romer and Romer (2010), Ramey (2011). There is a burgeoning literature on government expenditure multipliers. Ramey (2016) In the US context, it has been attempted to see whether government spending multipliers in good times differ from those in bad times. In plain language, the question is whether the size of a multiplier depends on the state of the

economy ([Auerbach and Gorodnichenko \(2013\)](#)). Using the technique of the local projection method due to [Jordà \(2005\)](#), the authors showed that a substantial shortage in the economy does not generate large multipliers; that is, multipliers are below one. In addition, they generally did not find large multipliers at the zero lower bound in the full sample. [Christiano, Eichenbaum, and Rebelo \(2011\)](#) used a DSGE model to show that the GSM can be very large (more than one) when the interest rate is constant, and rather small (less than one) when the Taylor rule prevails. [Corsetti, Meier, Müller, and Devereux \(2012\)](#) worked with a panel VAR of 17 OECD countries over the period 1975-2008 to show that the size of the multipliers depends on the economic environment, such as exchange rate regimes, public debt and the health of the financial system. Their results confirm the standard estimates of average fiscal multipliers, but with a large variation across economic environments. In particular, the exchange rate response to a spending shock depends on the exchange rate regime. Moreover, the size of the multiplier increases during financial crises. [Cogan, Cwik, Taylor, and Wieland \(2010\)](#) examined whether the package announced by euro area governments for 2009 and 2010 boosted GDP by more than one, i.e. whether the government spending multiplier was greater than one. Using five macroeconomic models with Keynesian features (wage and price rigidities), the paper found that at least four of the models indicated that the package would crowd out private consumption and investment. Here the government multiplier is below one. Only in one model for which the forward looking behaviour is ignored, that GSM is greater than one.

The fiscal multiplier studies for small states (particularly for the Caribbean) are rather meager ([Wright, Kallicharan, Mamingi, and Maynard \(2015\)](#), [Blake \(2013\)](#), [Kester and Belgrave \(2012\)](#), [Bangwayo-Skeete \(2014\)](#), and [Bynoe and Maynard \(2008\)](#), [Alichi, Sibata, and Tanyeri \(2021\)](#)), to name a few. [Wright et al. \(2015\)](#) explored the size and sign of fiscal shocks or multipliers (expenditures and tax) in Barbados using the structural VAR, the Bayesian VAR, and the DSGE models. It is shown that shocks to government spending have a small positive impact on output in the order of 0.15 at most across the models used. The tax impact is in general smaller than the spending. [Kester and Belgrave \(2012\)](#) used a SVAR to examine GSM for Barbados, Guyana, Jamaica as well as Trinidad and Tobago. While the size of GSM is small and does not vary too much across short term, peak and long term for Barbados in the order of 0.17, it is not the case for Jamaica where it is 0.11, 0.28 and 0.28, respectively for short term, peak and long term, for Guyana it is negative and small and for Trinidad and Tobago it goes from 0.18 in the short term to -0.57 in the peak time.

[Chinn \(2013\)](#) reviewed the literature on fiscal multipliers at the theoretical and empirical levels and found that the size of multipliers actually depends on the state of the economy, which is determined by the degree of slack in the economy, the state of the financial system and the conduct of monetary policy. For example, multipliers are generally larger in recessions than in booms. Multipliers in fixed exchange rate regimes are larger than those in flexible exchange rate regimes, reaching 1.5 (see [Born, Juessen, and Müller \(2013\)](#), [Ilzetzki, Mendoza, and Végh \(2013\)](#)).

A meta regression analysis on fiscal multipliers conducted by [Gechert and Will \(2012\)](#)

reveals that multipliers are model classes dependent. RBC generates multipliers lower than other models. DSGE New Keynesians and macroeconomic models lead to significantly different multipliers. There is no consensus yet as to the ideal size of multipliers.

Our paper departs from this methodology by focusing on Edgeworth substitutes (complements) in the utility function in a DSGE. To our knowledge, apart from the study of [Taufiq and Nana \(2018\)](#) for African economies, this paper is the first to present an explicit analysis of Edgeworth substitutes (complements) in the Caribbean and Latin America. The paper makes three contributions to the literature: one of a theoretical nature and two more empirical ones. First, at the theoretical level, it shows that GSM depends on country characteristics such as the degree of complementarity (substitutability) between private and public consumption and the elasticity of labour supply. More precisely, the paper derives the following results: (i) GSM is a decreasing function of the degree of complementarity between private and public consumption; (ii) the effect of the inverse of the Frisch elasticity on GSM is ambiguous, GSM is an increasing function of labour supply elasticity and (iii) GSM does not depend on the degree of procyclicality of government spending. Second, the paper adds to the limited experience of small states (especially in the Caribbean) with respect to the determination of the size of fiscal multipliers. In particular, it uses a DSGE model as opposed to the structural VAR used in almost all Caribbean studies. Third, since no consensus has yet been reached on the size of the GSM, this GSM adds to the pool of government expenditure multipliers. Section 2 develops the DSGE of interest in detail. Section 3 presents the results of the simulations and their interpretation based on the Bayesian method. Section 4 provides concluding remarks.

2 The model

In this section we describe the economic model. The DSGE model we use is an extended version of the one proposed by [Aguiar and Gopinath \(2007\)](#) and [Garcia-Cicco, Pancrazi, and Uribe \(2010\)](#). The main difference with the [Aguiar and Gopinath \(2007\)](#) model is that our framework includes complementarity/substitutability between private consumption and government spending in the utility function. We also include a number of additional structural shocks as discretionary public expenditure shocks and deterministic public expenditure shocks. The framework is a small open economy model ([Schmitt-Grohé and Uribe \(2003\)](#)). The economy consists of identical households with infinite lives, an unbounded number of competing firms producing the same homogeneous good with constant returns to scale. Households can trade a single asset on international financial markets. There are adjustment costs to the capital stock. There are external shocks to the economy that affect the real interest rate and total factor productivity. Time is discrete in this model.

2.1 The Production Sector

The representative firm produces an homogeneous final good denoted y_t with two inputs, capital denoted by k_t and labor denoted by l_t according to a constant returns to scale tech-

nology :

$$y_t = z_t k_t^\alpha (a_t l_t)^{1-\alpha} \quad (1)$$

in which t stands for time index, z_t and a_t are respectively the transitory and trend productivity shocks. $\alpha \in (0,1)$ is the elasticity of output with respect to capital. Trend shocks is specific to labor and define as $a_t = \gamma_t a_{t-1}$. Transitory and trend productivity shocks are captured by the following processes :

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}, \quad \text{with } |\rho_z| < 1, \epsilon_{z,t} \rightarrow iid(0, \sigma_z) \quad (2)$$

and

$$\gamma_t = \rho_\gamma \gamma_{t-1} + (1 - \rho_\gamma) \mu_\gamma + \epsilon_{\gamma,t}, \quad \text{with } \rho_\gamma < 1, \epsilon_{\gamma,t} \rightarrow iid(0, \sigma_\gamma) \quad (3)$$

where the random term has a normal distribution with zero mean. μ_γ is the long run growth of productivity. A realization of γ_t permanently influences a_t , output is then non stationary with a stochastic trend. We introduce the following transformation to denote its detrended variables : $\hat{x}_t = \frac{x_t}{a_{t-1}}$. The law of capital accumulation evolves according to :

$$k_{t+1} = (1 - \delta)k_t + i_t - \Phi(k_{t+1}, k_t) \quad (4)$$

where i_t is the investment flow, $\delta \in (0,1)$ denotes the rate of depreciation, and $\Phi(k_{t+1}, k_t)$ is the capital adjustment cost function assumed to verify $\Phi(0) = 0$ and $\Phi'(0) = 0$. The capital adjustment cost function takes a usual functional form : $\Phi(\cdot) = \frac{\phi}{2} \left(\frac{k_{t+1}}{k_t} - \mu_\gamma \right)^2 k_t$. ϕ is the parameter that governs the capital adjustment costs. We transform output into to it's detrended value denoted by \hat{y}_t :

$$\hat{y}_t = z_t \hat{k}_t^\alpha (\gamma_t l_t)^{(1-\alpha)} \quad (5)$$

Given \hat{w}_t and \hat{r}_t , the firm's program is to choose labor and capital in order to maximize profits, $\hat{\pi}_t$. The firm's profits are given by :

$$\hat{\pi}_t = \hat{y}_t - (1 - \tau_w) \hat{w}_t l_t - (1 - \tau_k) \hat{r}_t \hat{k}_t \quad (6)$$

where \hat{r}_t is the detrended real interest rate and \hat{w}_t is the detrended real wage rate, implies :

$$\hat{r}_t = \alpha \frac{\hat{y}_t}{(1 - \tau_k) \hat{k}_t} \quad (7)$$

$$\hat{w}_t = (1 - \alpha) \frac{\hat{y}_t}{(1 - \tau_w) l_t} \quad (8)$$

2.2 Government

We assume that the government adjusts transfers to balance its budget, so that public consumption is entirely financed by taxes:

$$\widehat{T}_t = \widehat{g}_t \quad (9)$$

The Government collects tax revenues from the economy by taxing income from labor and capital. The effective taxes rate are respectively τ_w ($\tau_w < 1$) and τ_k ($\tau_k < 1$). Total Government revenues are given by :

$$\widehat{T}_t = \tau_w \widehat{w}_t l_t + \tau_k \widehat{r}_t \widehat{k}_t \quad (10)$$

Finally by combining (9) and (10) we get :

$$\widehat{g}_t = \tau_w \widehat{w}_t l_t + \tau_k \widehat{r}_t \widehat{k}_t \quad (11)$$

As in Garcia-Cico (2012), government expenditures combine an endogenous component with a discretionary component. The fiscal policy is described by the following rule :

$$\ln(\widehat{g}_t/\widehat{g}) = \varphi_g \ln(\widehat{g}_{t-1}/\widehat{g}) - \varphi_y \ln(\widehat{y}_{t-1}/\widehat{y}) + z_{g,t} \quad (12)$$

where $\varphi_g \in (0, 1)$. Variables without a time subscript indicate steady-state values. Government purchases react with a lag to the level of economic activity. φ_y captures the degree of procyclicality of government spending. If $\varphi_g > 0$, government spending is pro-cyclical. If $\varphi_g < 0$, public spending is countercyclical. It is generally accepted that fiscal policy in emerging markets (or small states) tends to be pro-cyclical. In the previous relation, $z_{g,t}$ denotes the stochastic component of government expenditure, which follows an autoregressive process:

$$z_{g,t} = \rho_g z_{g,t-1} + \epsilon_{g,t} \quad (13)$$

Where $|\rho_g| < 1$. $\epsilon_{g,t}$ stand for a discretionary fiscal policy shocks.

2.3 The Household's program

The representative household consumes the private good and public consumption and maximizes the following utility function :

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t + \theta g_t, l_t) \quad (14)$$

where c_t , l_t and g_t are private consumption, labour and public consumption respectively¹ at time t . $\beta \in (0, 1)$ is the subjective discount factor. θ is the degree of substitutability (complementarity) between private and public consumption. $u(\cdot)$ is the current utility function,

¹We assume that g_t includes defence, public order and justice. Which can be seen as complementary to private consumption.

while $E(\cdot)$ is the expectation operator. Households treat public consumption as given and the total consumption of agents is a linear combination of private consumption and public consumption. This assumption is used by (Barro, 1981), (Aschauer & Greenwood, 1985) and more recently by Feve et al. (2013). As in Feve et al. (2013), θ represents the degree of substitutability between private and public consumption goods. To study the effect of g_t on the utility function, we need to discuss the parameter θ . When $\theta = 0$, public consumption has a negative income effect on labour supply; we are in a standard business cycle model. If $\theta = 1$, there is perfect substitution between private and public consumption. There is no effect on output and labour. When $\theta > 0$, public consumption substitutes private consumption. On the contrary, when $\theta < 0$, there is complementarity between public and private consumption.

While most of papers (Mendoza (1991), Garcia-Cicco et al. (2010) and Chang and Fernández (2013) among others) use the Constant Relative Risk Aversion (hereafter CRRA) because of their ability to improve the performances of small open economy models in reproducing some stylized facts, we adopt a separable preferences baseline:

$$u(c_t, l_t) = \left\{ \log(c_t + \theta g_t) - \omega \frac{l_t^{1+\nu}}{1+\nu} \right\} \quad (15)$$

where ν is the inverse of the Frisch elasticity of labour supply. Note that for the small open economy, public consumption crowds out private consumption for any high value of ν . ω captures the preference parameter affecting the disutility of labour. This functional form follows from Ganelli and Tervala (2009) and Feve et al. (2013) and allows the concept of Edgeworth substitutability to be captured and ensures greater analytical tractability. Intuitively, if private and public consumption are complementary (substitutes), an increase in public consumption increases (decreases) the marginal utility of private consumption, which limits (enhances) the negative welfare effect of the increase in public spending. If complementarity is sufficiently high, the effect on marginal utility may outweigh the negative wealth effect, leading to an increase in private consumption. However, in the presence of substitutability, the crowding-out effect of private consumption dominates.

The household's flow budget constraint is given by :

$$c_t + i_t + d_t \leq (1 - \tau_w)w_t l_t + (1 - \tau_k)r_t k_t + q_t d_{t+1} \quad (16)$$

In the above equation, d_t and q_t denote respectively the external debt² and the price of the net external debt due at time t .

First Order Conditions. For given values of initial foreign assets and physical capital, the representative households supply labour and decide on the level of consumption in a competitive market and purchase one-period bonds, maximising lifetime utility (15) subject

²Note that small states cannot borrow on the international capital market because of their high debt levels.

to the capital accumulation equation (4) and the resource constraint (16) and taking prices and fiscal policy as given. The first-order conditions can be written as follows³ :

$$\widehat{\lambda}_t = \frac{1}{\widehat{c}_t^*} \quad (17)$$

$$\widehat{\lambda}_t(1 - \tau_w)\widehat{w}_t = \omega l_t^v \quad (18)$$

$$\widehat{\lambda}_t = \beta \gamma_t^{-1} q_t E_t \widehat{\lambda}_{t+1} \quad (19)$$

$$\widehat{\lambda}_t \left[1 + \phi \left(\gamma_t \frac{\widehat{k}_{t+1}}{\widehat{k}_t} - \mu_\gamma \right) \right] = \beta \gamma_t^{-1} E_t \widehat{\lambda}_{t+1} \times \left[1 - \delta + r_t + \phi \gamma_{t+1} \frac{\widehat{k}_{t+2}}{\widehat{k}_{t+1}} \left(\gamma_{t+1} \frac{\widehat{k}_{t+2}}{\widehat{k}_{t+1}} - \mu_\gamma \right) - \frac{\phi}{2} \left(\gamma_{t+1} \frac{\widehat{k}_{t+2}}{\widehat{k}_{t+1}} - \mu_\gamma \right)^2 \right] \quad (20)$$

where $\widehat{c}_t^* = \widehat{c}_t + \theta g_t$. These equations show that the marginal rate of substitution of leisure for consumption must be equal to the wage, the Euler equation for capital, the Euler equation for foreign bonds. In addition, the transversality condition for physical capital and foreign assets must hold. Substituting the FOC (17) and (6) into the FOC (18), we obtain the condition that equates the marginal rate of substitution between consumption and leisure to the opportunity cost of an additional unit of leisure:

$$\omega l_t^{1+v} c_t^* = (1 - \alpha)(1 - \tau_w)\widehat{y}_t \quad (21)$$

Equilibrium. The market clearing condition on the goods market can be writes :

$$\widehat{y}_t = \widehat{c}_t + \widehat{i}_t + \widehat{g}_t \quad (22)$$

Now that we have calculated optimal decisions for both households and firms, we can derive the equilibrium of the model. This is done by adding together the decisions of both economic agents. The definition of equilibrium is then as follows:

Definition 1. A competitive equilibrium for this economy is a vector of allocation, $\left\{ \widehat{c}_t, \widehat{k}_t, l_t, \widehat{d}_t, \widehat{g}_t \right\}_{t=0}^{\infty}$ and factor price $\{w_t, r_t\}_{t=0}^{\infty}$ given $\{a_t\}_{t=0}^{\infty}$ and a vector of fiscal policy parameters $\{\tau_w, \tau_k, \theta\}_{t=0}^{\infty}$ such that : i) The optimization problem of the consumer is satisfied, ii) Given prices for capital and labor, the first-order conditions of the firm hold, iii) the feasibility constraint of the economy is satisfied.

³We have to note that the following variables, $\widehat{c}_t, \widehat{k}_t, \widehat{b}_t$ and \widehat{w}_t are measured in detrended units.

2.4 Financial friction

As [Uribe and Yue \(2006\)](#) and [Neumeyer and Perri \(2005\)](#), we assume that the small open economy faces a debt-elastic interest-rate premium, such that the gross interest rate paid is given by :

$$\frac{1}{q_t} = 1 + r^* + \psi \left[e^{\frac{d_{t+1}}{a_t} - \bar{d}} - 1 \right] + e^{(s_t - 1)} \quad (23)$$

where r^* and \bar{b} are the external interest rate (assumed to be constant) and the steady state of normalised debt, respectively. ψ captures the elasticity of the borrowing rate to changes in debt. s_t captures an exogenous stochastic country premium shock. We assume that the rest of the world is willing to lend any amount to the domestic economy at the rate r_t . Lending to this economy is risky because of the risk of default. We assume that the country spread, s_t is driven by an two exogenous process : the TFP shocks, z_{t+1} and $z_{g,t+1}$, the government spending shocks :

$$s_t = -\eta_z E_t z_{t+1} + \eta_g E_t z_{g,t+1} + \epsilon_{s,t+1} \quad (24)$$

and $\epsilon_{s,t+1}$ captures the country spread shock with zero mean and variance σ_s^2 . η_z and η_g are positive parameters describing the sensitivity of spreads to future productivity. Factors other than productivity shocks may influence the country risk premium. For example, country risk may also be positively correlated with changes in government expenditure, with large deficits being associated with higher country risk.

2.5 Analytical results

To compute the long run Government spending Multiplier (GSM), we need to characterize the deterministic steady state. Using the first order condition (20), (21) :

$$\psi l^{1+\nu} = \frac{(1-\alpha)(1-\tau_w)\hat{y}}{c^*} \quad (25)$$

$$1 = \hat{\beta} \left(1 - \delta + \alpha \frac{\hat{y}}{(1-\tau_k)\hat{k}} \right) \quad (26)$$

where⁴ $\hat{\beta} = \beta \mu_\gamma^{-1}$, and the equilibrium definition (22) and (4) :

$$\hat{y} = \hat{c} + \hat{i} + \hat{g} \quad (27)$$

$$\hat{i} = (\gamma - (1 - \delta)) \hat{k} \quad (28)$$

⁴We consider that in a balanced Growth Path $\gamma = \mu_\gamma$.

The steady state of this economy is solution to the previous system of equations. From equation (26) and (28) we can solve for capital-output ratio :

$$\frac{\widehat{y}}{\widehat{k}} = \frac{(1 - \widehat{\beta}(1 - \delta))(1 - \tau_k)}{\widehat{\beta}\alpha} \quad (29)$$

and for investment ratio

$$i_y = \frac{\widehat{\beta}\alpha(\gamma - (1 - \delta))}{(1 - \tau_k)(1 - \widehat{\beta}(1 - \delta))} \quad (30)$$

where $i_y = \widehat{i}/\widehat{y}$. Total differentiation of (27) yields

$$d\widehat{c} = (1 - i_y)d\widehat{y} - d\widehat{g} \quad (31)$$

By totally differentiating the equilibrium conditions (25), we have :

$$\psi(1 + \nu)l^{1+\nu}dl = \frac{(1 - \alpha)(1 - \tau_w)}{c^*} \left(d\widehat{y} - \frac{\widehat{y}}{c^*} [d\widehat{c} + \theta d\widehat{g}] \right) \quad (32)$$

Taking into account that $dl = \frac{l}{\widehat{y}}d\widehat{y}$ and using (8), (31) into (32) and the identity $1 = c_y + i_y + g_y$, we get :

$$(1 - i_y + \nu(c_y + \theta g_y)) d\widehat{y} = (1 - \theta)d\widehat{g} \quad (33)$$

where $c_y = \widehat{c}/\widehat{y}$, $g_y = \widehat{g}/\widehat{y}$.

Definition 2. *The long run Government spending Multiplier.* The long run fiscal multiplier denoted $\mathcal{M}(\theta) = \frac{d\widehat{y}}{d\widehat{g}}$ is the ratio of a change in output to the change in government spending that causes it.

An increase in \widehat{g} could increase or decrease \widehat{y} depending on the relative value of θ and ν . The transmission channels of the fiscal multiplier are wealth and substitution effects. The increase in public spending reduces agents' wealth due to the increase in taxes, which leads to a reduction in private consumption. This leads to a labour-leisure substitution effect. The strength of this effect depends on the complementarity/substitutability between private and public consumption. The previous results are therefore fairly familiar in the traditional findings of the fiscal policy literature when, $\theta = 0$ and ν . and $\epsilon_w = 0$.

Proposition 1. *With the previous assumptions*

1. *The long run Government Spending Multiplier (GSM) is :*

$$\mathcal{M}(\theta) = \frac{1 - \theta}{1 - i_y + \nu(c_y + \theta g_y)}$$

2. *The government expenditure multiplier is a decreasing function of the degree of complementarity between private and public goods, θ .*

3. *The government expenditure multiplier is a decreasing function of the inverse of the Frisch elasticity.*

4. *The government expenditure multiplier does not depend on φ_y .*

Proof. To prove this proposition we derive, $\mathcal{M}(\theta)$ respected to θ

$$\frac{\partial \mathcal{M}(\theta)}{\partial \theta} = - \frac{(1+\nu)(c_y+g_y)}{\left((c_y+g_y)+\nu(c_y+\theta g_y)\right)^2} < 0$$

The higher the substitutability between private and public consumption, the lower the GSM. However, if government spending is complementary, an increase in \hat{g} reduces the labour supply of households. Equation (33) describes the government expenditure multiplier, which can provide interesting insights. First, several cases can be considered in terms of the degree of complementarity (substitutability), θ :

- When $\theta > 1$, the GSM is negative, $\mathcal{M}(\theta) < 0$,
- When $\theta = 1$, Private consumption is a perfect substitute for government spending. In this case, government spending has no effect on output, $\mathcal{M}(\theta) = 0$,
- When $\theta = 0$, the GSM is positive, in this case, $\mathcal{M}(\theta) = \frac{1}{(1+\nu)c_y+g_y}$,
- When $\theta \leq \theta^*$ (where $\theta^* = \frac{1-(1+\nu)c_y+g_y}{(1+\nu g_y)}$, for $\mathcal{M}(\theta) = 1$), the GSM is more than one, $\mathcal{M}(\theta) \geq 1$.

Second, we characterise the lower bound of θ for which the marginal utility of consumption is positive. This value is given for which

$$\frac{\partial u}{\partial \hat{c}} \geq 0 \tag{34}$$

holds. The degree of substitutability verifies :

$$\tilde{\theta} \geq - \frac{c_y}{g_y} \tag{35}$$

and given $\tilde{\theta}$ the GSM is more than one. The figure 1 below shows the range of interesting values of θ for which we can draw the effects of the government expenditure multiplier, and calls for the following comments: If $\frac{\partial^2 \mathcal{M}}{\partial \theta^2} > 0$, then $\mathcal{M}(\theta)$ is convex in θ . We can note some interesting properties of the model : $\lim_{\theta \rightarrow \infty} \mathcal{M}(\theta) = -\frac{1}{\nu g_y}$, i.e. the GSM has a lower limit as θ goes to infinity. Furthermore, the previous relationship makes it clear that $\mathcal{M}(\theta)$ has an asymptote given by $-\frac{1}{\nu} \left(1 + (1+\nu) \frac{c_y}{g_y}\right)$, which is the maximum value of theta compatible with a multiplier greater than one. Appendix A gives additional information on the value of $\mathcal{M}(\theta)$ taking into account the frich elasticity and the degree of complementarity.

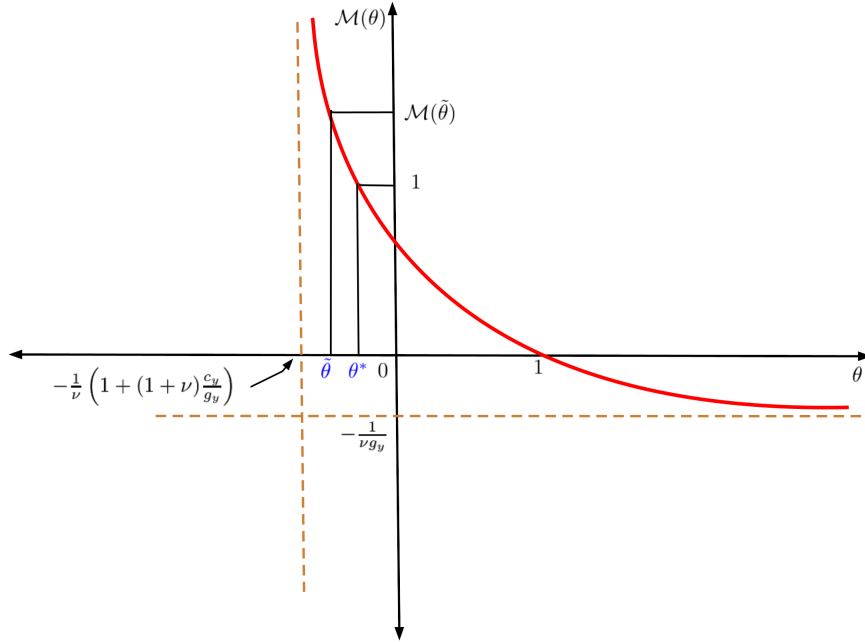


Figure 1: Government Spending Multiplier and the degree of complementarity (substitutability). As θ goes to ∞ , $\mathcal{M}(\theta)$ tends asymptotically towards to $-\frac{1}{\nu g_y}$. Similarly, the GSM has a vertical asymptote which is represented by the maximum limit

3 Simulation and results

The purpose of this section is to use data to obtain values for the parameters of the model. We estimate and calibrate the model for several small countries, in particular for Caribbean economies. The time unit in the model is the quarter. We use detrended quarterly data to estimate the theoretical framework. We confront the predictions of our theoretical model with the data, based on a panel of small states, mainly Caribbean countries (Jamaica, Barbados, Trinidad and Tobago and the OECS countries). The theoretical model predicts that the sign and magnitude of the multiplier will depend on the degree of complementarity of goods. Our objective is to estimate the short-run and long-run GSM.

Given the highly non-linear equilibrium conditions, the policy function that determines the present and future values cannot be solved analytically. We therefore solve the model numerically. Given the problems associated with the estimation of theta, we have opted for the Bayesian method. We have calibrated some parameters of the model.

3.1 Data

The dataset consists of three time series⁵: output, private and public consumptions. As a contribution to previous studies, we use public investment.

Observations are quarterly and cover the period 1993Q1 to 2019Q4 for Jamaica and 2006Q1 to 2021Q4 for Barbados. Note that data on government expenditure and private consumption are not available at quarterly frequency. Therefore, in order to harmonise the frequencies, an interpolation using the *Denton-Cholette* method was necessary. We have not considered the trend in the data for several reasons. (i)The sample is limited, making it impossible to observe any trend effects on government expenditure. (ii)Open economy models are usually estimated with nonstationary data. To extract the non-stationary component, we use the Hodrick-Prescott filter with a smoothing parameter equal to 1600.

Calibration. As our data sample does not allow us to estimate all the underlying parameters of the model, we choose a combination of calibration and estimation. Formally, we split the parameter vector Θ into two parts: $\Theta_1 = \{\beta, \alpha, \delta, \bar{b}, \tau_k, \tau_w, \omega, \nu, \mu_g\}$ contains some structural parameters that may be poorly identified and should therefore be calibrated. The discount factor β is set to 0.989 which gives a quarterly interest rate of 1.53%. The value of α is taken from the national accounts data. The value for this parameter is close to 0.356. Another parameter to be calibrated is the depreciation rate. The value of δ corresponds to the ratio of investment to output. We found that the corresponding rate is $\delta = 0.050$. The inverse of the Frisch elasticity of labour supply, ν , is set to ensure a value close to 2 for the preference parameter, ω , and 23% for the working time, l . These different parameters form the basic calibration valid for all Caribbean countries. The steady state growth rate, μ_g , is the average of output growth, which is 3% (see table 1).

Table 1: Harminised calibration parameters according to quarterly data

Parameters	Definition	Value
β	Discount factor	0.989
α	Technological parameter	0.356
δ	Capital depreciation rate	0.050
τ_k	Capital income tax rate	0.250
τ_w	Labor income tax rate	0.150
ω	Preferences parameter	2.000
ν	Inverse of the Frisch elasticity	4.063
μ_g	The steady state growth rate	0.003

We set the steady state of normalised debt, \bar{b} , equal to the average debt ratio for each country in the data, $\bar{b} = 122\%$ for Jamaica and $\bar{b} = 126\%$ for Barbados. In addition, the

⁵The data come from the Central Bank of Jamaica, the Statistical Institute of Jamaica, the Central Bank of Barbados, the IMF and the World Bank. We have used capital expenditure as a proxy for public expenditure for Jamaica. These data were not easily available in Barbados.

government expenditure ratio is fixed at 18.80% and 33.74% for Jamaica and Barbados respectively (see table 2).

Table 2: Specific calibration parameters according to quarterly data

Parameters	Definition	Jamaica	Barbados
\bar{b}	Debt share	1.222	1.260
τ_k	Capital income tax rate	0.250	0.250
τ_w	Labor income tax rate	0.150	0.250
\hat{g}	Government spending ratio	0.295	0.337

Estimation. Following the procedure described in [An and Schorfheide \(2007\)](#), the Bayesian method is used to estimate the other parameters of the model. The set of parameters to be estimated are the following : $\Theta_2 = \{ \phi, \psi, \varphi_g, \varphi_y, \theta, \eta_z, \eta_g, \rho_z, \rho_g, \rho_\gamma, \sigma_z, \sigma_g, \sigma_\gamma, \sigma_s \}$. As noted, there are three observables, y_t , c_t and g_t . The choice of priors is a very important step in Bayesian estimation. We have chosen the best priors for all the economies studied. In this paper, we have a limited amount of information on which to base the priors. This may explain why some of the DSGE modelling for Caribbean economies uses the calibration method instead. Regarding the priors, some of them are selected from the [Uribe and Yue \(2006\)](#) studies. As [Garcia-Cicco et al. \(2010\)](#) noted, the importance of the magnitude of the debt sensitivity, ψ , has important implications for the dynamics in the model. As a result, our priors, ψ , can take values significantly larger than zero and follow a uniform distribution. Similarly, capital adjustment costs, ϕ , can be very high in small open economies due to after-sales service costs. Consequently, we use a uniform distribution for capital adjustment costs with a value of 6.70.

To estimate the model, we used a sample of 1,000,000 draws and obtained an acceptance rate of about 33% for Jamaica and Barbados, which is quite significant. The univariate Markov Monte Carlo chain (MCMC) diagnostic test of [Brooks and Gelman \(1998\)](#), the prior and posterior distributions of the parameters and the Blanchard-Kahn conditions are used to assess the goodness of fit of the Bayesian estimators. The analysis was performed using 1 000 000 Metropolis Hastings draws. The two chains for each parameter should evolve in a similar way⁶. We also ran the test suggested by [Iskrev \(2010\)](#), which checks the identification and sensitivity of the parameters. The results of the test indicate that all parameters are identifiable in the neighbourhood of the posteriors.

Table 3 shows the posterior means of the model parameters and the 95% confidence interval. Focusing on our estimates of θ , note that the posterior of the degree of complementarity (substitutability) is negative ($\theta = -0.591$) for Jamaica, suggesting that public spending is an Edgeworth complement for Jamaica. For the Barbadian economy, on the other hand, the coefficient θ is 0.221. This indicates Edgeworth complementarity. Focusing on the coefficient φ_y , our estimated results show that fiscal policy tends to be procyclical, as the coefficients φ_y are equal to 0.459 (Jamaica) and 0.469 (Barbados). This result is comparable to that obtained

⁶Simulation data available on request.

by Feve et al. (2013) for the US economy.

Table 3: Priors and posteriors for the parameters

		Prior distributions			Posterior distributions		Posterior distributions	
		Shape ¹	Mean	Std ²	Mean	Conf.intervals	Mean	Conf.intervals
STRUCTURAL PARAMETERS								
Degree of procyclicality	ϕ_y	\mathcal{B}	0.500	0.027	0.459	[0.413;0.506]	0.469	[0.423;0.515]
public consumption coefficient	ϕ_g	\mathcal{B}	0.960	0.022	0.732	[0.700;0.765]	0.539	[0.502;0.573]
Degree of complementarity	θ	\mathcal{U}	0.000	0.090	-0.596	[-0.978;-0.199]	0.260	[0.170;0.350]
Debt sensitivity	ψ	\mathcal{U}	0.004	0.010	0.003	[0.002;0.005]	0.005	[0.004;0.005]
Capital adjustment cost	ϕ	\mathcal{G}	6.500	0.500	6.664	[5.826;7.461]	6.719	[5.902;7.529]
PROPAGATION PROCESS								
Risk-premium TFP shock	η_z	\mathcal{G}	0.270	0.029	0.232	[0.189;0.273]	0.273	[0.224;0.322]
Risk-premium public consumption shock	η_g	\mathcal{G}	0.250	0.029	0.226	[0.187;0.264]	0.432	[0.391;0.472]
TFP AR parameter	ρ_z	\mathcal{B}	0.815	0.016	0.861	[0.841;0.881]	0.816	[0.790;0.842]
Public consumption AR parameter	ρ_g	\mathcal{B}	0.450	0.012	0.466	[0.446;0.486]	0.412	[0.392;0.430]
Trend AR parameter	ρ_γ	\mathcal{B}	0.900	0.012	0.892	[0.870;0.915]	0.899	[0.879;0.920]
Std TFP	σ_z	$\mathcal{IN}\mathcal{V}\mathcal{G}$	0.009	inf	0.003	[0.003;0.004]	0.001	[0.001;0.002]
Std public consumption	σ_g	$\mathcal{IN}\mathcal{V}\mathcal{G}$	0.025	inf	0.057	[0.049;0.064]	0.154	[0.130;0.178]
Std trend	σ_γ	$\mathcal{IN}\mathcal{V}\mathcal{G}$	0.095	inf	0.008	[0.007;0.009]	0.002	[0.001;0.002]
Std spread shock	σ_s	$\mathcal{IN}\mathcal{V}\mathcal{G}$	0.016	inf	0.003	[0.003;0.004]	0.027	[0.001;0.002]

¹ This column shows prior distributions according to which the acronyms respectively indicate : \mathcal{B} as Beta distribution, \mathcal{G} as gamma distribution, \mathcal{U} Uniform as distribution and $\mathcal{IN}\mathcal{V}\mathcal{G}$ designates the inverse-Gama distribution.

² Std stand for Standard deviation.

3.2 Fiscal multiplier implications

In this section, we present the estimation results in order to assess the theoretical model. We then analyse the impact of a discretionary shock to public spending.

3.2.1 The long run GSM

This section summarises the impact of government consumption on the economy in the long run. The GSM, $\mathcal{M}(\theta)$, is constructed according to equation (33). Table 4 shows the multipliers for output, consumption and investment (respectively $\mathcal{M}(\theta)$, $\mathcal{M}(\theta)_c$ and $\mathcal{M}(\theta)_i$) based on the mean estimates of the posterior distribution. The estimated GSM is less than one. Comparing the output multipliers of the different economies, table 4 shows that the output multiplier is higher in Jamaica than in Barbados, where public and private consumption are Edgeworth substitutes. Despite the fact that the structural parameters of these economies are similar, the public expenditure multiplier is very different. firstly, the use of capital expenditure in Jamaica may explain this difference. Second, Jamaica started to implement a so-called Structural Adjustment Programme since 2013. Consequently, public investment should be at the centre of countercyclical growth policies.

We observe a crowding out effect on private consumption, as the persistently high taxes have a significant negative wealth effect on consumers, suggesting a Ricardian effect. On the other hand, this effect is offset by the positive effect on investment for all economies considered, i.e. 0.583 and 0.184 for Jamaica and Barbados respectively. However, the investment multiplier for Barbados remains low, in contrast to Jamaica.

Table 4: Estimated fiscal multiplier

	$\mathcal{M}(\theta)^1$	$\mathcal{M}(\theta)_c$	$\mathcal{M}(\theta)_i$
Jamaica	0.737 (0.045)	-0.845 (0.053)	0.583 (0.049)
Barbados	0.221 (0.029)	-0.963 (0.049)	0.184 (0.043)

¹ Standard errors in parentheses. $\mathcal{M}(\theta)_c$ and $\mathcal{M}(\theta)_i$ are respectively private consumption and investment multipliers.

With regard to the results obtained for developing countries using similar methods, the results for the fiscal multipliers are in line with those for the expenditure multipliers for the United States in [Feve et al. \(2013\)](#).

3.2.2 The discretionary public spending shock

The second experiment concerns the effect of a temporary shock to public consumption (discretionary shock, see Fig. 2). The response to this shock is an increase of output for all periods, but only a small one, close to zero for Jamaica after 20 quarters. As far as the economy of Barbados is concerned, it is the consumption of the government that is more effective. These different results can be explained by the fact that Jamaica is implementing a structural adjustment programme due to excessive debt. This finding is consistent with recent literature showing that increasing government spending leads to limited output growth in the Caribbean ([Alichu et al., 2021](#); [Blake, 2013](#); [Kester & Belgrave, 2012](#)).

With a peak in the five quarters following the shock, the model generates a positive consumption response, and unlike in the previous case, the Ricardian effect does not affect consumption in the short run. Moreover, the consumption response is comparable between the model and the VAR.

Temporary government spending shocks have a positive impact on investment in Barbados, but a negative impact on investment in Jamaica within five quarters.

Overall, the effects of temporary public expenditure shocks are negligible for all economies. This result highlights the fact that it is public investment shocks that are favourable to economic growth.

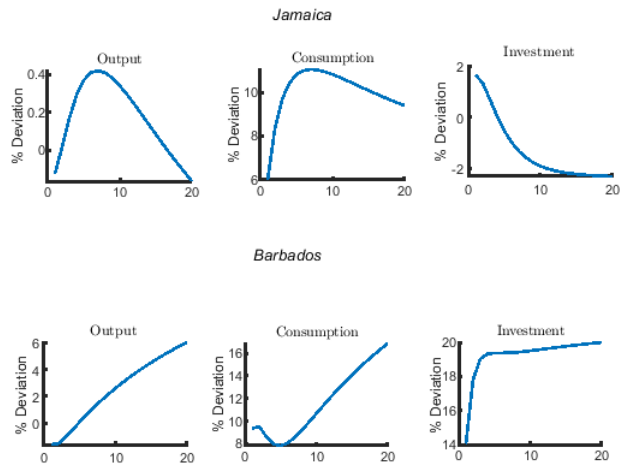


Figure 2: Cumulative impulse responses after a discretionary shock to public spending

4 Conclusion

In this paper, we develop a small open economy DSGE model with a non-stationary productivity process and financial frictions to analyse the size and sign of the government expenditure multiplier. We first consider a framework for the analytical derivation of the long-run government expenditure multiplier. We show that the government spending multiplier is an increasing function of the Edgeworth complementarity coefficient. We also estimate a DSGE model using Bayesian techniques for the sample period 1993Q1 to 2019Q4 and 2006Q1 to 2019Q4 for Barbados, Jamaica, Trinidad and Tobago and the group of ECCU members, respectively. The results show that the sign of the GSM differs across countries. However, we have provided preliminary results that deserve to be discussed and extended to other Caribbean countries.

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APPENDIX

A GSM and labor supply elasticity

In order to quantitatively validate the size of the fiscal multiplier, we extend our analysis to include the role of the Frisch elasticity. In the previous sections we have shown that the sign and size of the public expenditure multiplier is also closely related to the elasticity of household labour supply. Using equation (33) and differentiating with respect to ν , this implies that

$$\frac{\partial \mathcal{M}(\theta)}{\partial \nu} = -\frac{(1-\theta)(c_y + \theta g_y)}{((c_y + g_y) + \nu(c_y + \theta g_y))^2}$$

The sign of the derivative is clearly ambiguous; it depends on θ . We can get an idea of the profile of the derivative by plotting it in the three-dimensional plane. The important parameters are, of course, θ and ν . The parameters are taken from the literature, we assume that $c_y = 0.70$ and $g_y = 0.20$.

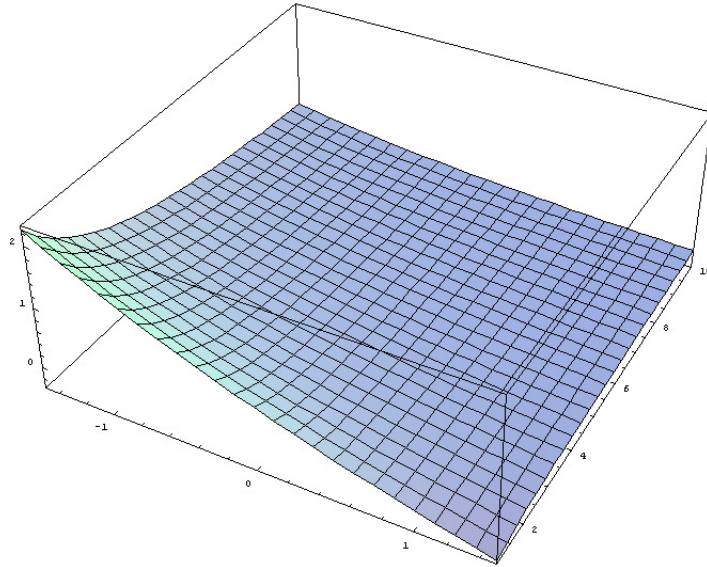


Figure 3: Multiplier profile