Measuring Extreme Cross-Market Dependence for Risk Management: The Case of Jamaican Equity and Foreign Exchange Markets

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This version: 13 May 2005

Abstract

The aim of this study is to examine the extreme cross-market dependence between the Jamaican foreign exchange and equity markets and its implications for risk managers. Extreme value *copulas* are used to model the joint distribution of extreme US/Jamaica foreign exchange and equity returns. For didactic purposes, this paper also employs a graphical approach to estimating value-at-risk (VaR) using extreme value theory (EVT). Daily data from 01/01/1992 to 01/07/2003 is used to model the tail behaviour of the foreign exchange and equity returns. Univariate EVT-VaR tail quantiles at the 95 per cent level are generated for use by risk managers. In the bivariate case, the paper finds a high frequency of a foreign exchange-equity co-boom relative to a co-crash. This strong *right tail dependence* arises from the existence of arbitrage opportunities between cross-listed stocks in the Jamaica Stock Exchange (JSE) and Trinidad & Tobago Stock Exchange (TTSE) caused by significant depreciations in the US/Jamaica foreign exchange rate. The existence of the arbitrage channel is supported by a robustness check, which indicates the *absence* of right tail dependence when the composite JSE index is substituted with an index that excludes the cross-listed stocks.

1.0 Introduction

Recent episodes of financial crises in emerging economies have highlighted the need for more sophisticated internal market risk control systems as well as the appropriate external (regulatory) controls. Consequently, there has been a tremendous growth in financial risk modelling by both internal and external risk managers. The value-at-risk (VaR) model has quickly developed into the benchmark approach among practitioners and regulators for computing composite risk measures and capital allocation. VaR reflects the maximum potential loss in the value of a portfolio over a fixed horizon and for a given probability. In the prudential context, VaR provides an estimate of the risk capital that is required to cover portfolio losses over a fixed holding period. The probability level used in calculating the VaR is typically very small so as to capture only extreme market fluctuations. Although many financial systems in the developed economies have adopted these requirements, most emerging financial markets are still resisting its regulatory implementation. Nevertheless, the obvious usefulness of summarizing market risk in a single summary statistical measure has prompted many of the financial institutions in emerging markets to begin to measure their market risks[1] using the VaR model.

Large price shocks in financial markets correspond to extreme events such as crashes in equity, bond and foreign exchange markets. The Basel Committee on Banking Supervision (1996) recommends that the VaR measure for bank portfolios that are exposed to extreme fluctuations in market prices be based on the 99th percentile of a single-tailed confidence interval, for a ten-day holding period. As such, the market risk capital of a bank must be adequate to cover losses in its trading portfolio in 99 per cent of the occurrences over a ten-day horizon. A bank's trading portfolio is, by its nature, impacted by more than one risk factor. An important problem faced by risk managers is the modelling of joint distributions of the different risk factors. Additionally, recent research has uncovered major flaws with the typical use of the G*aussian* assumption in modelling. This paper uses *extreme value copulas* to address these problems. A copula may be defined as a function that links univariate marginal distributions to their joint multivariate distribution. Copulas were introduced in the late 1950's, mainly through the research of Sklar (1959) to examine probabilistic metric spaces, and have recently become popular in the modelling of joint distributions of separate financial markets.[2]

The aim of this study is to examine a unique phenomenon concerning the extreme cross-market dependence between the Jamaican foreign exchange and equity markets and its implications for risk managers. For didactic purposes, this paper also employs a graphical approach to estimating VaR using extreme value theory (EVT).

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[1] That is, the risk of losses on a financial portfolio due to adverse price fluctuations.
[2] See Costinot, Roncalli and Teïletche (2000) and Rockinger and Jondeau (2001).

In the context of probability theory, extreme price shocks occur in the tails of the price distribution. This section of the probability distribution, which captures extraordinary market events, is the focus of the VaR model. Obviously, the accurate generation of VaR estimates is critical for the proper management of risk and capital adequacy. Nonetheless, there are a number of competing statistical models for estimating VaR. These may be classified into parametric, nonparametric and semi-parametric groups. The most popular parametric approach is the variance-covariance method based on the assumption of a conditional normal price return process, such as in GARCH models.[3] This method involves frequent updates to the variance-covariance matrix of returns in order to incorporate information on recent extreme movements in prices. The main drawback of this approach is that financial returns series are known to be leptokurtic or "fat tailed." Hence, assuming a normal distribution process will lead to a significant under-prediction of extreme events.[4] The Basel Committee recommends that the capital requirement on market risks be calculated as the individual bank's internal VaR measure, times a "multiplication factor" of 3 to account for the potential misspecification[5] of the loss distribution.[6] However, the choice of the multiplication factor is subject to much debate.[7]

The typical nonparametric approach to computing VaR relies on the process of historical simulation. For example, the historical simulation (HS) approach utilizes a rolling window of historical market prices to forecast the probability distribution of future losses (or profits). This method does not rely on *a priori* assumptions about the data generation process. Instead, it assumes that the probability distribution of returns is constant regardless of the sample window used in computing VaR. This has proved to be a limiting assumption, however, as the infrequency of extreme market events typically result in a high variability of the VaR estimates.

Contrary to the variance-covariance and historical simulation methods, EVT focuses on the tails of the distribution rather than the centre of the distribution. The tails are the primary interest of risk managers. According to EVT, the limiting distribution of the extreme returns is unrelated to the actual data generation process. As a result, knowledge of the true underlying distribution for the returns series is not relevant for VaR estimation. Given the smaller reliance on distribution assumptions, this semi-parametric approach has been found to offer more reliable VaR estimates for prudent financial risk management. Historically, EVT has been applied to hydrology, engineering, insurance and other fields involving extreme observations. Recent research on the application of EVT to financial risk management

[3] See, for example, JP Morgan RiskMetrics methodology.
[4] See Danielsonn and de Vries (1997).
[5] The basic reason for using this multiplication factor is to account for the fact that the normality assumption, which is the basis for many of the market risk models, is widely accepted as unrealistic.
[6] Stahl (1997) uses Chebyshev's inequality to arrive at the correction factor of 3 under the standard normal distribution.
[7] See, for example, Danielsonn and de Vries (1997).

includes: Falk, Hüsler and Reiss (1994), Longin (1996), Reiss and Thomas (1996), Danielsonn and de Vries (1997), Embrechts, Klüppelberg and Mikosch (1997), Straetmans (1998), McNeil (1999), Jondeau and Rockinger (1999) and McNeil and Frey (2000).

Recall that the value-at-risk for a financial portfolio measures the maximum possible financial loss from holding the portfolio over a fixed horizon for a given level of significance (or *quantile*). More formally, the VaR is the p^{th} quantile of the distribution function F, such that $VaR_p = x_p = F^{\leftarrow}(p) = \inf\{x \in \Re : F(x) \geq p\}$, where $F^{\leftarrow}$ denotes the generalised inverse of F and $0 < p < 1$. However, the risk manager's use of VaR has been criticized on two grounds. First, in a series of influential papers including Artzner, Delbaen, Eber, and Heath (1997, 1999) and Dalbaen (2000), it is argued that the non-Gaussian VaR is not a *coherent* risk measure and, hence, does not represent an accurate aggregation of risks across portfolios. A risk measure ρ is coherent if it satisfies the following four axioms: (i) *monotonicity*: if $X \geq 0$ then $\rho(X) \leq 0$; (ii) *subadditivity*: $\rho(X_1 + X_2)) \leq \rho(X_1) + \rho(X_2)$; *positive homogeneity*: for $\lambda \geq 0$, $\rho(\lambda X) = \lambda\rho(X)$; and, *translation invariance*: for $a \in \Re$, $\rho(a + X) =$ $\rho(X) - a$. The quantile-based VaR measure does not satisfy the subadditive property. This means that the sum of the individual VaRs for the financial sub-portfolios is *less* than the VaR of the total portfolio. Another shortcoming of the VaR measure is that it only yields the frequency estimate on the upper bound of losses. Thus, VaR does not provide information on the severity of an extreme loss, given that a loss exceeding the VaR upper bound has occurred.

Artzner et al (1997, 1999) propose the *expected shortfall* measure, otherwise called the "tail conditional expectation," as a coherent substitute to VaR. This risk measure provides an estimate of the potential size of the loss that exceeds VaR. This expected loss size, or the expected shortfall (S), given that VaR_p is exceeded, is represented as: $S_p = E[X \mid X > VaR_p]$.[8] It can be shown that the expected shortfall risk measure is subadditive, in that, the merging of sub-portfolios cannot increase risk.

1.1 Cross-Market Arbitrage Opportunities

This paper analyses the interdependency between the extreme observations in the Jamaican foreign exchange and equity markets. The typical channel in which an extreme linkage between these two markets would arise is through the portfolio effect. For example, if there were a sustained depreciation of the US/Jamaica Dollar foreign exchange (FX) rate, *ceteris paribus*, investors would substitute Jamaica Dollars for US Dollars, thus drawing liquid funds from the stock exchange (or bond market). The opposite would occur in the case of a sustained appreciation in the rate.

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[8] Formally, the expected shortfall measure is obtained from the relationship: $\int_{-\infty}^{t} x[f(x)/F(t)]dx$.

However, another unique channel exists, directly as a result of the thinness in the local financial (ie, equity and foreign exchange) market. This channel arises from the existence of arbitrage opportunities between cross-listed stocks in the Jamaica Stock Exchange (JSE)[9] and Trinidad & Tobago Stock Exchange (TTSE) caused by significant depreciations in the US/Jamaica Dollar foreign exchange rate. The arbitrage process has been well documented by local and Trinidadian investment analysts and the press. For example, according to an article in a major local newspaper:[10]

> "Yesterday, the JSE index closed up by nearly 1 000 points, fuelled by stocks which are listed locally, and on the Trinidad stock market. With the devaluation, the prices of these stocks in Trinidad are worth more in Jamaican dollar terms. This price differential has created arbitrage opportunities –driving the demand for stocks which can then be traded in Trinidad at a much higher price."

However, the slide in the value of the Jamaica Dollar vis-à-vis its US counterpart must be large enough (that is, at least equal to the transactions costs) for investors to take advantage of the deviation in relative prices. In this case, a significant increase in the US/Jamaica Dollar exchange rate will simultaneously trigger a proportional rise in the JSE index given the new demand for the cross-listed stocks, *ceteris paribus*. However, if the rate of depreciation is small, investors would not find it profitable to conduct arbitrage, resulting in an insignificant impact on the JSE index. Further, the share of market capitalisation for the cross-listed stocks must be large enough for the cross-market arbitrage phenomena to significantly influence extreme movements in the composite JSE index. As reported in the local press and a Central Bank publication,[11] occurrences of significant *cross-market arbitrage* was particularly evident during the episodes of significant currency devaluation in the first half of 2003. As at 30 June 2003, only 6 of the 40 firms listed on the JSE were cross-listed on the TTSE. However, the cross-listed firms accounted for 65 per cent of market capitalisation. Therefore, it is expected that any extreme co-dependence between the foreign exchange rate and the price and/or volume of the cross-listed stocks will be *relatively significant in the right tails* of the foreign exchange and JSE index returns series.

As a robustness check of the cross-market arbitrage channel, the extreme cross-market dependencies are re-computed by replacing the JSE index with the "All Jamaica" JSE index.[12] The companies listed in the All Jamaica JSE index are selected according to two criteria: 1. It excludes all foreign companies that are not incorporated under the Jamaica Companies Act; 2. It includes only the 15 highest ranked stocks according to ordinary volumes, number of transactions and number of days traded within a $T-3$-year moving window. As shown in Tables IA-IIIA (in Appendix A), the stocks that are cross-listed in both the JSE and TTSE stock exchanges are *not included* in the All Jamaica JSE index.

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[9] www.jse.org.jm

[10] See "J$ dips again: Stock market gaining from devaluation," *Jamaica Business Observer* (May 16, 2003).

[11] See Bank of Jamaica, *Quarterly Monetary Policy Report* (June 2003).

[12] The JSE computation of the All Jamaica index began at the end of April 2000. Thus, the All Jamaica index and the composite JSE index are equivalent prior to this date.

Thus, the estimated extreme co-dependence parameter for the right tails of the US/J$ foreign exchange returns and the All Jamaica JSE index returns should be below the matching parameter in the case when the composite JSE index is used, in order to provide empirical support for the cross-market arbitrage channel.

The rest of section 1 is concerned with the modelling of extreme cross- market dependence. Section 2 describes EVT and section 3 illustrates its use for risk managers using foreign exchange and equity returns series. Section 4 discusses and implements the maximum-likelihood process for computing the EVT-copulas. A robustness check of the arbitrage channel is conducted in section 5. Section 6 provides some brief concluding remarks.

1.2 Extreme Cross-Market Dependence

Intuitively, the correlation between two financial series would provide essential information in computing the probability of a co-crash in both markets. However, it can be shown that extreme value dependence is unrelated to the shape of the extreme values (i.e., the tails) of the univariate marginal distribution functions. For example, two heavy tailed Pareto distribution functions, though uncorrelated, may be asymptotically dependent. Multivariate extreme value theory (MEVT) involves the direct computation of the probability of a co-crash or extremal spillovers,[13] without the use of the correlation coefficient.

With the purpose of exploring the probability of a co-crash, let (X,Y) denote a pair of asset returns and let $\omega(=0,1,2)$ denote the number of simultaneous market crashes. Extreme values, or crashes, are observed when $X>s$ or $Y>t$. The conditional probability that both markets crash simultaneously given that at least one market crashes is expressed as:[14]

$$P\{\omega=2 \mid \omega\geq 1\}=\frac{P\{X>s,Y>t\}}{1-P\{X\leq s,Y\leq t\}}=\frac{P\{X>s\}+P\{Y>t\}}{1-P\{X\leq s,Y\leq t\}}-1. \qquad [1]$$

A *dependency measure* may be defined, which is equivalent to the conditional probability given in [1]. The dependency measured is provided using probability theory to obtain the conditional expected number of market crashes given by:

$$E\{\omega \mid \omega\geq 1\}=\frac{P\{X>s,Y\leq t\}+P(X\leq s,Y>t)+2P\{X>s,Y>t\}}{1-P\{X\leq s,Y\leq t\}}$$

$$=\frac{P\{X>s\}+P\{Y>t\}}{1-P\{X\leq s,Y\leq t\}}=P\{\omega=2 \mid \omega\geq 1\}+1 \qquad [2]$$

How can risk managers construct an appropriate dependency measure? Risk managers are typically concerned with determining the joint distribution of the risk factors that impact their financial

[13] See Straetmans (2000) and Hartman, Straetmans and de Vries (2001).
[14] The probability of a crash is mapped into the first quadrant.

portfolio. Linear measures of correlation, such as the Pearson correlation coefficient, have proved inaccurate in the case of approximating the joint distribution of extreme returns because of the absence of an appropriate dependence measure for the extreme returns as well as non-linear dependence processes.[15] For example, even if the Pearson coefficient generates a value of zero, two series can nevertheless be dependent.[16] It has been found that the dependencies among financial markets are different for the stable and extreme observations located in the centre and the tails of the distribution, respectively. That is, the statistical properties of financial market prices are different during stable and crisis periods. If these differences are not taken into account, the generated risk measure is unreliable.[17] Thus, the standard correlation coefficient of extreme values between the two markets is an unreliable measure of market dependency because the underlying distribution is unknown.

1.2 Modelling Dependence using Copulas

As will be shown in section 2, EVT allows for the identification of the univariate marginal distribution functions without knowing the underlying distributions. Given this advantage, EVT has become the standard resource for describing extreme financial events. Accordingly, multivariate extreme value theory (MEVT) offers a solution for the accurate description of the joint dependence structure of extreme series through the use of *copulas*. Straetmans (1999), Stărică (1999), Longin and Solnik (2000) and Costinot, Roncalli and Teïletche (2000) are recent examples of the use of MEVT to model the dependence between financial markets.

In the bivariate case, suppose the distributions functions F_1 and F_2 are continuous, then there exists a unique function, $C:[0,1]^2 \to [0,1]$, with standard uniform marginals $F_1(X_1)$ and $F_2(X_2)$ so that:

$$\begin{aligned} C(u,v) &= P\{X_1 \le u, X_2 \le v\} \\ &= H\left[F_1^{\leftarrow}(u), F_2^{\leftarrow}(v)\right] \\ &= C[\mathrm{F}_1(u), F_2(v)] \end{aligned} \qquad [3]$$

where $F_i^{\leftarrow}(s) = \inf\{x \in \Re \mid F(x) \ge s\}$ and H is a 2-dimensional joint distribution function. This is the standard 2-dimensional copula representation of the distribution of the random vector $[X_1, X_2]^T$.[18] Importantly, the copula satisfies the following three properties: 1. $C(u,v)$ is increasing in u and v; 2.

[15] The linear (Pearson) correlation coefficient is expressed as: $\rho(X,Y) = \dfrac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{\mathrm{Var}(Y)}}, -1 < \rho < 1.$

[16] See Embrechts McNeil and Straumann (1999) for a discussion on the pitfalls associated with correlation.

[17] See, for example, Costinot, Roncalli and Teïletche (2000) and Embrechts, Lindskog and McNeil (2001).

[18] See Sklar (1983).

$C(0,v)=C(u,0)=0,\ \ C(1,v)=v,\ \ C(u,1)=u$; 3. $\forall u_1,u_2,v_1,v_2$ in $[0,1]$ such that $u_1 < u_2$ and $v_1 < v_2 : C(u_2,v_2)$ - $C(u_2,v_1)$ - $C(u_1,v_2)+C(u_1,v_1) \geq 0$. Statistically, there exists *upper tail dependence* if:

$$P\left\{X_2 > F_2^{\leftarrow}(v) \mid X_1 > F_1^{\leftarrow}(u)\right\} \geq P\left\{X_2 > F_2^{\leftarrow}(v)\right\},$$

which can be re-written as:

$$\frac{P\left\{X_2 > F_2^{\leftarrow}(v) \mid X_1 > F_1^{\leftarrow}(u)\right\}}{P\left\{X_1 > F_1^{\leftarrow}(u)\right\}} \geq P\left\{X_2 > F_2^{\leftarrow}(v)\right\},$$

or $$P\left\{X_2 > F_2^{\leftarrow}(v) \mid X_1 > F_1^{\leftarrow}(u)\right\} \geq P\left\{X_1 > F_1^{\leftarrow}(u)\right\} P\left\{X_2 > F_2^{\leftarrow}(v)\right\}.$$ [19]

In the case $u=v$, the coefficient of upper tail dependence of $(X_1,X_2)^T$ may be defined as:

$$\begin{aligned}\lambda(u) &= \lim_{u\to 1^-} P\left\{X_2 > F_2^{\leftarrow}(u) \mid X_1 > F_1^{\leftarrow}(u)\right\} - P\left\{X_2 > F_2^{\leftarrow}(u)\right\} \\ &= \lim_{u\to 1^-} P\left\{X_2 > F_2^{\leftarrow}(u) \mid X_1 > F_1^{\leftarrow}(u)\right\}\end{aligned}$$

If $\lambda \in (0,1]$, X_1 and X_2 are asymptotically dependent and if $\lambda = 0$ (ie, $C(u,v)=uv$), X_1 and X_2 are asymptotically independent.[20]

This paper considers two examples of copulas:[21] the Gumbel-McFadden model and the Hüsler-Reiss[22] model. The bivariate Gumbel-McFadden copula is given by the function:[23]

$$C_\delta(u,v) = \exp\left\{-[(-u)^\delta + (-v)^\delta]^{1/\delta}\right\},\ \ u,v<0 \qquad [4]$$

with density:

$$C_\delta{}'(u,v) = C_\delta(u,v)(uv)^{\delta-1}\left(-\left((uv)^\delta + (-v)^\delta\right)^{2(1/\delta-1)} + (\delta-1)\left((-u)^\delta + (-v)^\delta\right)^{1/\delta-2}\right) \qquad [5]$$

where the dependence parameter is $\delta \in [1,\infty]$ so that $C_1(u,v)=\exp(u)\exp(v)$ and $C_\infty = \exp\left(\min\{u,v\}\right)$. A value of $\delta = 1$ implies independence and $\delta = \infty$ reflects complete dependence.[24] The bivariate Hüsler-Reiss copula is given by the function:

$$C_\delta(u,v) = \exp\left\{-\Phi\left(\delta^{-1} + \frac{1}{2}\delta(u-v)\right)\exp(-v) - \Phi\left(\delta^{-1} + \frac{1}{2}\delta(v-u)\right)\exp(-u)\right\},\ u,v<0 \qquad [6]$$

with density:

$$C_\delta{}'(u,v) = C_\delta(u,v)\left(\Phi\left(\delta^{-1} + \frac{1}{2}\delta(v-u)\right)\Phi\left(\delta^{-1} + \frac{1}{2}\delta(u-v)\right)\exp-(u+v) + \delta\Phi'\left(\delta^{-1} + \frac{1}{2}\delta(v-u\right)\frac{\exp-(u)}{2}\right) \qquad [7]$$

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19 According to Joe (1997), if the dependence function is larger than the product copula, $C(u,v)=uv$, then there exists positive quadrant dependence.

20 In the bivariate case: $\lambda(u) = \lim_{u\to 1^-} (1-2u+C(u,u)) \big/ (1-u)$.

21 See Reiss and Thomas (1997).

22 See Hüsler and Reiss (1989).

23 The thresholds u and v will hereafter represent log values.

24 See Galambos (1987).

where the dependence parameter is $\delta \in [0,\infty]$ and Φ is the univariate standard normal distribution function.

2.0 Extreme Value Theory

According to the theorems of Fisher and Tippett (1928**)** and Gnedenko (1943), regardless of the specific distribution of a series, the appropriately scaled maxima converges to one of three possible limit laws (parametric distributional forms), under certain conditions. A standardised form of the three limit laws is called the generalised extreme value distribution. Additionally, by the theorems of Balkema and de Haan (1974) and Pickands (1975), the distribution function of the excesses above a high threshold converges to the generalised Pareto distribution. This section outlines these two key theorems that underpin extreme value theory.

2.1 The Extreme Value Distribution

Suppose the univariate sequence $\{X_1,\ldots,X_n\}$ represents independent, identically distributed (*iid*) random variables, such as daily changes in the logarithm of asset prices, with a (marginal) distribution function F. Suppose that $\max(X_1,\ldots,X_n) = -\min(-X_1,\ldots,-X_n) \equiv M_n$, then the probability that the maximum asset returns, M_n, falls below x is

$$\begin{aligned} P(M_n < x) &= P(\max(X_1,\ldots,X_n) < x) \\ &= \mathrm{P}(X_1 < x,\ldots,X_n < x) \\ &= F_n(x). \end{aligned} \quad [8]$$

An analog to the Central Limit Theorem, extreme value theory provides conditions whereby the distribution of the location-scale normalisation of the maximum, irrespective of F, is non-degenerate as $n \to \infty$.

Theorem 1 *Let* X_n *be a sequence of iid random variables. If there exist norming constants* $a_n > 0$*,* $b_n \in \Re$ *and some non-degenerate distribution function* $G(x)$ *such that*

$$P\left\{\frac{M_n - a_n}{b_n} \le x\right\} = F^n(b_n x + a_n) \xrightarrow{d} G(x) \quad [9]$$

then $G(x)$ *belongs to one of the three standard extreme value distributions.*[25]

Type I (Gumbel): $G(x) = \exp(-e^{-x}), x \in \Re,$ *(if* F *has light tails);*

Type II (Fréchet): $G(x) = \begin{cases} \exp(-x^{-\alpha}), x > 0 \\ 0 \quad , x \le 0 \end{cases}, \ \textit{for } \alpha > 0$ *(if* F *has heavy tails);*[26]

[25] For proofs of the theorem see the cited references.

[26] A distribution function has heavy tails if it varies regularly at infinity:
$\lim_{n\to\infty} \frac{F(-nx)}{F(-n)} = \lim_{n\to\infty} \frac{1-F(nx)}{1-F(n)} = x^{-\alpha}, x > 0, \alpha > 0.$

Type III (Weibull): $G(x)=\begin{cases}\exp(-(-x)^{\alpha}), & x<0\\ 1, & x\geq 0\end{cases}$, *for* $\alpha>0$, *(if the support of* F is *finite)*.

The three distributions are subsumed in the following generalised extreme value (GEV) distribution

$$G_{\xi,\mu,\sigma}(x)=\begin{cases}\exp[-(1+\xi x)]^{-1/\xi} & \text{if } \xi\neq 0\\ \exp[-\exp(-x)] & \text{if } \xi=0\end{cases}, \qquad [10]$$

where $1+\xi x>0$ and $\xi\in\Re$ is a *shape parameter*. The standard representations can be restated by setting $\xi=0$ for the Gumbel distribution, $\xi=\alpha^{-1}>0$ for the Fréchet distribution and $\xi=-\alpha^{-1}<0$ for the Weibull distribution.

Given G_{ξ}, EVT finds the conditions on F so that the normalised sample maxima converges to $G(x)$ for appropriate sequences $\{a_n\}$ and $\{b_n\}$. In this case, F is said to be in the maximum domain of attraction of $G(x)$, i.e. $F\in MDA(G_{\xi})$. This condition holds, in general, for all continuous distributions. Financial times series commonly satisfy the condition $F\in MDA(G_{\xi})$ where $\xi>0$, implying that F is in the main domain of attraction of the heavy-tailed Fréchet distribution.[27] As shown by Gnedenko (1943), for $\xi>0$, $P(X>x)=x^{-1/\xi}L(x)$, where $L(x)$ is a slowly varying function.[28] Distributions in $MDA(G_{\xi})$ where $\xi<0$ or $\xi=0$ correspond to short-tailed and thin-tailed distributions, respectively, are not suitable for modelling extreme losses in financial time series.

2.2 The Generalised Pareto Distribution

Let X be a random variable with the distribution function F. Consider the excesses above a very high threshold u. Define the conditional probability that the excess losses over u, ie $X-u$, is less than y, given $X-u>0$, as

$$F(x\mid X>u)=P(X-u\leq y\mid X>u)=\frac{F(u+y)-F(u)}{1-F(u)}\equiv F_u, \qquad [11]$$

for $0\leq y<x_F-u$ and the right endpoint, $x_F=\sup\{x\in\Re:F(x)<1\}\leq\infty$.

Theorem 2 *For a certain class of distributions, there exists a positive scaling function* $\beta(u)$ *such that*

$$\lim_{u\to x_F}\ \sup_{0\leq y<x_F-u}\left|F_u(y)-H_{\xi,\beta(u)}(y)\right|=0 \qquad [12]$$

where

[27] This distribution class includes the Pareto, the Student-t, the Cauchy, and the log-gamma among others.

[28] That is, for all $\lambda>0$, $\lim_{x\to\infty}\frac{L(\lambda x)}{L(x)}=1$.

$$H_{\xi,\beta(u)} = \begin{cases} [1-(1+\xi \frac{y}{\beta})]^{-1/\xi} & if\ \xi \neq 0 \\ 1-\exp(-\frac{y}{\beta}) & if\ \xi = 0 \end{cases}, \qquad [13]$$

and

$$y \geq 0 \text{ if } \xi \geq 0\,;\ 0 \leq y \leq -\beta/\xi \text{ if } \xi < 0.$$

2.3 Application of Extreme Value Theory to Measures of Extreme Risk

Following from section 2.2, by setting $x = u + y$, then $F(x) = (1-F(u))F_u(y) + F(u)$. Hence, the probability of falling below a certain minimum threshold, $F(x)$, can be estimated with the following function, known as the *tail estimation formula*:

$$\begin{aligned} \hat{F}(x) &= \frac{N_u}{n}[1-(1+\frac{\hat{\xi}}{\hat{\beta}}(x-u))^{-1/\hat{\xi}}] + [1-\frac{N_u}{n}] \\ &= 1-\frac{N_u}{n}(1+\frac{\hat{\xi}}{\hat{\beta}}(x-u))^{-1/\hat{\xi}} \end{aligned}, \qquad [14]$$

where $F_u(y) \approx H(y)$ and $F(u) \approx (n-N_u)/n = m/n$, and where n is the total number of observations and N_u is the number of observations above the threshold u. Inverting the tail estimation formula yields the associated quantile estimate for a given probability $p > F(u)$:

$$\hat{x}_p = u + \frac{\hat{\beta}}{\hat{\xi}}[(\frac{n}{N_u}(1-p))^{-\hat{\xi}} - 1]\,. \qquad [15]$$

The expected shortfall is expressed, relative to VaR_p as:

$$\hat{S} = \hat{x}_p + E[X-\hat{x}_p \mid X > \hat{x}_p]\,, \qquad [16]$$

where $E[X-\hat{x}_p \mid X > \hat{x}_p]$ is the mean of the exceedances of $F_{x_p}(y)$ over x_p. If $\xi < 1$, the *mean excess function* is $(\beta + \xi(x_p - u))/(1-\xi)$, where $(\beta + \xi(x_p - u)) > 0$ and, hence, the expected shortfall estimate may be explicitly calculated as:

$$\hat{S} = \hat{x}_p + \frac{\hat{\beta} + \xi(\hat{x}_p - u)}{1-\hat{\xi}} = \frac{\hat{x}_p}{1-\hat{\xi}} + \frac{\hat{\beta} - \hat{\xi}u}{1-\hat{\xi}}\,. \qquad [17]$$

3.0 Data Description and Statistical Modelling of the Tail Behaviour

The objective of this section is to model the tail behaviour of the J$/US$ daily returns and the JSE index daily returns to obtain estimates of their extreme quantiles. Daily data on the J$/US$ selling rate were obtained from the External Sector Unit of the BOJ, and daily stock market data, including the JSE composite index (which includes the cross-listed stocks) and the All Jamaica index, were downloaded from the JSE website. Both series cover the period ranging from 01/01/1992 to 01/07/2003. All returns series, r_t, were generated using the continuous compounding formula $r_t = \ln(P_t) - \ln(P_{t-1})$, where P_t represents the price series at time t, and "$\ln$" is the natural logarithm. The exchange rate and JSE composite index return series are illustrated in Figures 1 and 2, respectively. Figure 3 presents a cross plot of the returns series. The occurrences of co-dependence in the foreign exchange and equity markets are evident in all four quadrants of the figure. However, there is no discernible evidence of a greater co-dependence in any of the quadrants relative to the other three.

Figure 1. FX Returns

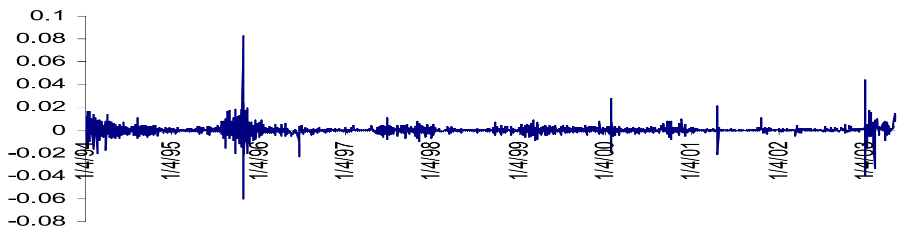


Figure 2. JSE Returns

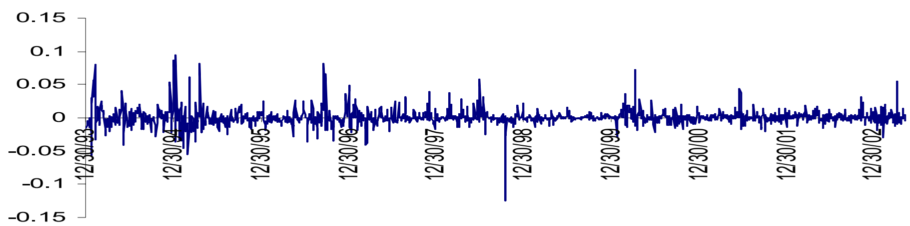
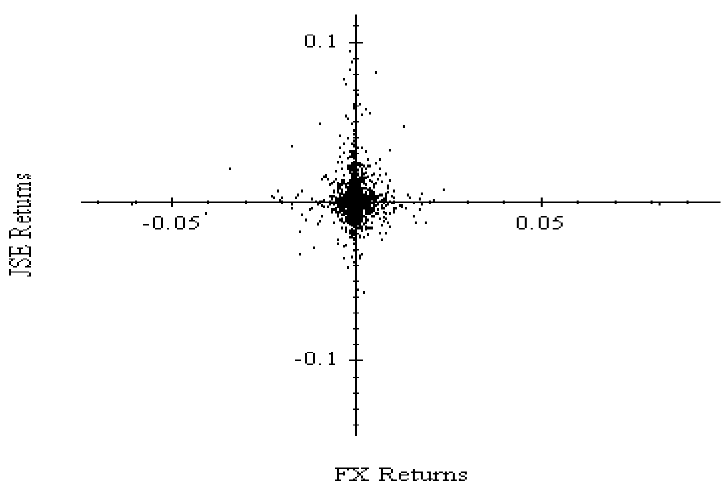


Figure 3. Scatterplot of Foreign Exchange and JSE Index Returns



The standard correlation matrix for the exchange rate and index is shown in Table 1. Consistent with the asset substitution theory, the exchange returns and the stock returns are negatively correlated. However, the use of average correlations does not account for the asymmetric and non-linear time series characteristics associated with extreme events. The correlation structure is likely to be significantly altered during market crises. This paper uses copulas to capture all of the information on the extreme dependence between these financial markets.

Table 1. Standard Correlation Matrix

	JUS	JSE
JUS	100%	-1.58%
JSE	-1.58%	100%

3.1 Normality Testing

The summary statistics for the daily logarithmic J$/US$ exchange and JSE index returns are presented in Table 2. The skewness and kurtosis statistics are both positive, with their magnitudes indicating that both series have long and "fat" right tails. The Jarque-Bera test statistics for normality confirm that the null hypothesis of a normal distribution is rejected in both cases at the one per cent level of significance.

Table 2. Summary Statistics

	J$/US$	JSE Index
Mean	0.000214	0.000661
Median	0.000125	-0.000151
Maximum	0.082950	0.094210
Minimum	-0.060337	-0.124471
Std. Dev.	0.004256	0.011768
Skewness	1.566795	1.246452
Kurtosis	105.1451	21.35350
Jarque-Bera	860279.1	28260.01
Probability	0.000000	0.000000
Observations	1977	1977

3.2 Q-Q Plots

The Q-Q plot is also an important graphical measure in order to determine the appropriate limit law of the series'. In other words, the Q-Q plot may be used as a pre-test for the domain of attraction. Let $F^{\leftarrow}(x)$ denote the p^{th} quantile of F, and $F_n^{\leftarrow}(x)$ denote the inverse of the empirical distribution function. A simple method to check the limit type of a series is to plot $F^{\leftarrow}(x)$ against $F_n^{\leftarrow}(x)$. If $X_{(i)}$ represents the ascending order statistics, where $i=1,\ldots,n$, then the Q-Q plot (the graph of quantiles) is defined by the set of points: $\{X_{(i)}, F^{\leftarrow}(i/n)\}$. The theoretical cdf inverse of the Gumbel is $x_p = -\ln(-\ln(i/n))$. Given that the calculation of the inverse of the Gumbel does not require parameter estimation, it can be simply plotted against the observed X_p.[29] If $F_n(x)$ is Gumbel distributed, then its quantiles should match with those of $F(x)$, thus producing a linear Q-Q plot. If the upper tail area of the Q-Q plot is convex (slopes upward), then the limit distribution is Weibull; if the slope is concave, then the limit distribution is Fréchet. Figures 4 and 5 confirm that the empirical distributions functions of the FX returns and the JSE returns, respectively, are in the domain of attraction of the Fréchet.

Figure 4. Gumbel (Right Tail) Plot of Frechet - Log(FX Returns)

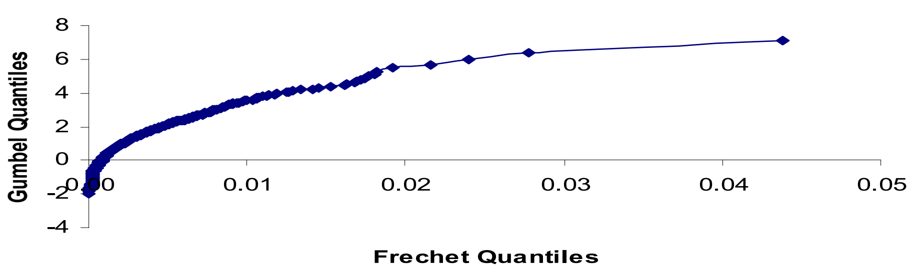
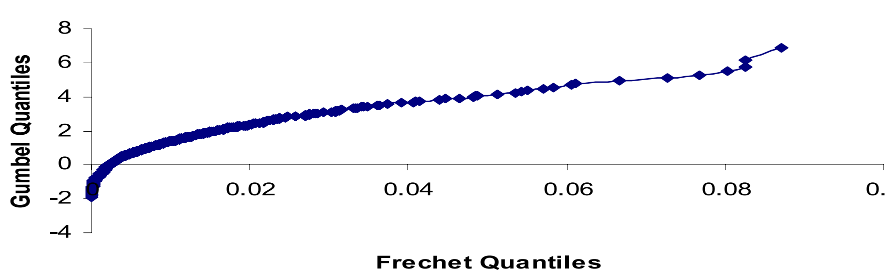


Figure 5. Gumbel (Right Tail) Plot of Frechet - Log(JSE Returns)



[29] See Gumbel (1958).

3.3 Choosing the Threshold: Graphical Tools

The threshold, u, must be chosen high enough to satisfy Theorem 2. Two popular graphical methods for estimating the threshold value are the mean excess function and the Hill plot. The drawback of the graphical approach, however, is if the chosen threshold is too high then the estimates will exhibit high variability due to the paucity of observations for estimation in the tail. On the other hand, if the threshold choice is too low, then observations from the centre of the distribution will lead to biased indexes. Thus, there exists an important trade-off between variance and bias.

3.3.1 The Mean Excess Function Plots

The sample mean excess plot provides an important graphical measure to arrive at the optimal choice of the threshold. This method is especially useful given that no standard algorithm for choosing an acceptable threshold exists. The sample mean excess plot is expressed as: $\{(u, e_n(u)) : X_{1:n} < u < X_{n:n}\}$, where $X_{1:n}$ and $X_{n:n}$ are the first and n-th ascending order sample statistics. The sample mean excess function is defined by:

$$e_n(u) = \frac{\sum_{i=1}^{n}(X_i - u)I_{\{X_i > u\}}}{N_u}. \qquad [18]$$

This function depicts the sum of the exceedances over u, divided by the number of exceedances over u, i.e., $N_u = \sum_{i=1}^{n} I_{\{X_i > u\}}$, where $I_{\{X_i > u\}}$ is an indicator variable that equals 1 if $X_i > u$, and 0 otherwise. The threshold u is chosen from the mean excess function at the start of the part on the graph where the function is approximately linear. The mean excess functions are depicted in Figures 6 and 7 for the FX returns and JSE returns, respectively. The "thresholds" are on the x-axis and the "excess over threshold" on the y-axis. These figures suggest that the values of the thresholds, u, are higher in the case of extreme losses compared to extreme gains from holding equity[30] and US Dollars. The values of u chosen for the right and left tails of FX returns are 0.006 ($N_u = 105$) and 0.07 ($N_u = 48$), respectively. The values of u chosen for the right and left tails of JSE returns are 0.017 ($N_u = 107$) and 0.02 ($N_u = 44$), respectively.

[30] That is, a value weighted portfolio of all stocks listed on the JSE.

Figure 6a. Right Tail Mean Excess Function – FX Returns

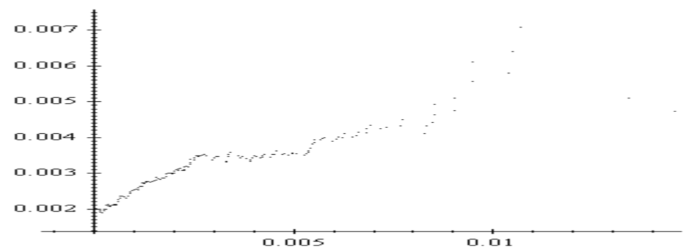


Figure 6b. Left Tail Mean Excess Function – FX Returns

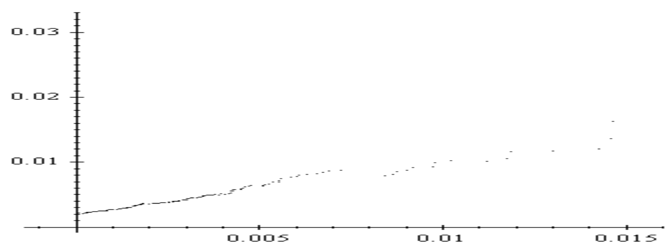


Figure 7a. Right Tail Mean Excess Function – JSE Returns

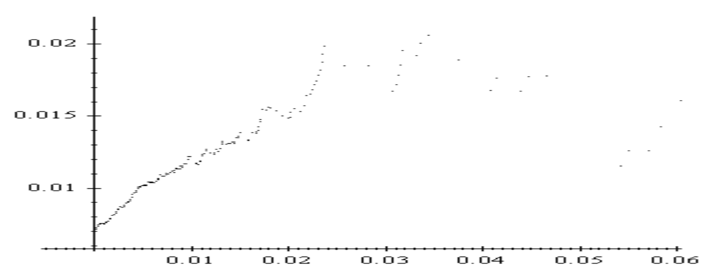
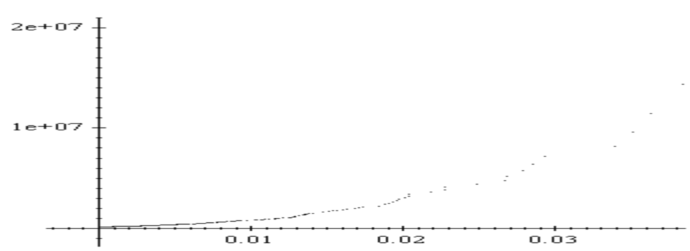


Figure 7a. Left Tail Mean Excess Function – JSE Returns



3.3.2 Tail Index Estimation: The Hill Estimator

This paper employs a semi-parametric approach to estimate the tail index. In contrast to the parametric approach, this approach avoids the strong assumption that the extreme observations are distributed exactly GEV. Instead, the semi-parametric approach requires only that the extreme values more or less approximate the GEV distribution. This is enough to satisfy $F \in MDA(G_{\xi})$.

Let $F(-x) = x^{-1/\xi} L(x)$ for $-x \leq -u$, then the conditional distribution function is $F(-x \mid X \leq -u) = (x/u)^{-1/\xi}$ and the conditional density is $f(-x \mid X \leq -u) = (1/\xi)(x/u)^{((-1/\xi)-1)}(1/u)$. To obtain the Hill estimator, the maximum likelihood principle is applied by taking the log of the conditional density and differentiating the first-order condition with respect to $1/\xi$. Then x is replaced by $\sum_{i=1}^{m} X_i$ for $X_i > u$. Hence, the Hill (1975) estimator of the tail index, ξ, based on $m+1$ upper order statistics is:

$$H_{m,n} = \frac{1}{m} \sum_{i=1}^{m} \ln \frac{X_i}{X_{m+1}} (= \hat{\xi}) \qquad [19]$$

where m is the number of upper order statistics that is used in estimation. The estimator, $H_{m,n}$, is consistent for ξ if m increases such that:

$$m/n \to 0 \quad \text{as} \quad n \to +\infty .$$

That is, m is selected by minimizing the sample *mean squared error* (MSE). The Hill estimator is also asymptotically normal such that:

$$\sqrt{n}(H_{m,n} - \xi) \to \mathrm{N}(0, \xi^2).$$

This method of estimating the tail index is optimal only for the Fréchet case ($\xi > 0$).

The Hill plot may be expressed as: $\{(m, H_{m,n}),\ 1 \leq m \leq n-1\}$. The number of upper order statistics, m, is chosen in the region of the plot where the tail index, ξ, is stable. Figure 8 and 9 show the Hill plots for the FX returns and the JSE returns, respectively.

Figure 8a. Hill Plot of Right Tail - Log(FX Returns)

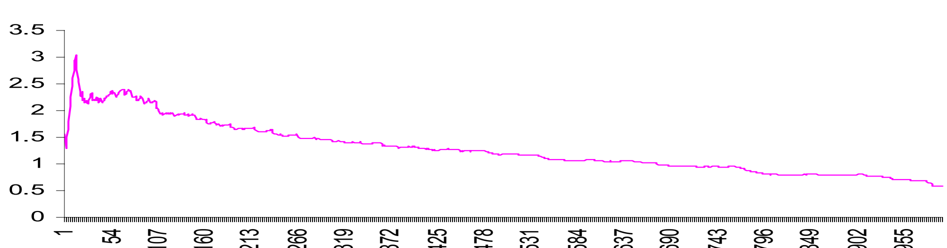


Figure 8b. Hill Plot of Left Tail - Log(FX Returns)

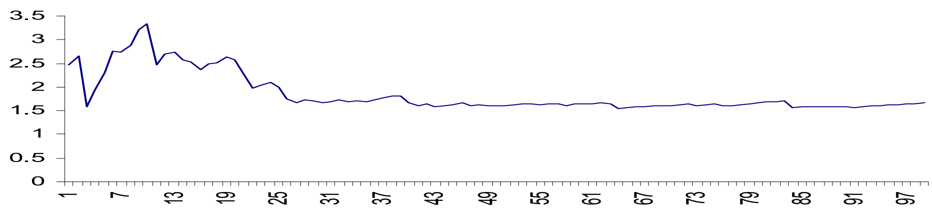


Figure 9a. Hill Plot of Right Tail - Log(JSE Returns)

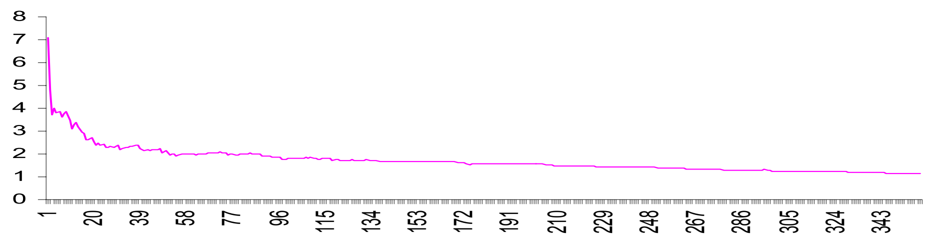
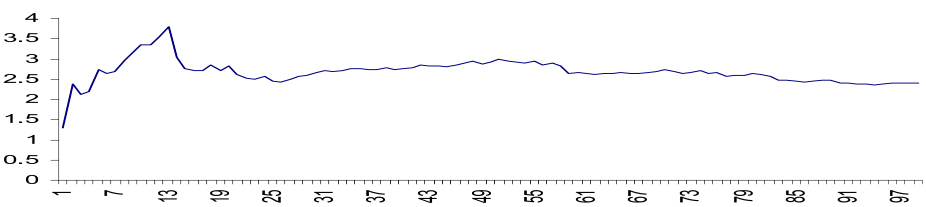


Figure 9b. Hill Plot of Left Tail - Log(JSE Retuns)



3.4 Quantile Estimation

Recall from section 3 that the VaR_p estimate is the p^{th} quantile of the distribution function F, such that $P\{X \geq -VaR_p\} = F(-x_p \mid X \leq -u) * F(X \leq -u) \equiv F(-x_p) = x_p^{-1/\xi} L(x_p) = p$. Equating this definition with the conditional distribution function $F(-x \mid X \leq -u) = (x/u)^{-1/\xi}$ yields:

$$x_p = u\left[\frac{F(u)}{p}\right]^{\xi}. \quad [20]$$

Furthermore, replacing u with X_{m+1}, and $F(u)$ with $\tilde{F}(X_{m+1}) = X_{m+1}^{-1/\xi} L(X_{m+1}) = (n - N_u)/n = m/n$ provides the quantile estimator:

$$\hat{x}_p = X_{m+1}\left[\frac{(m/n)}{p}\right]^{\hat{\xi}}. \quad [21]$$

The quantile estimators for the maxima and minima of the log FX and JSE returns are shown in Figures 10a-11b for $p = 0.95$. Consistent with the Hill plot, the VaR estimate is chosen in the region where the quantile plot is stable. Appendix B provides the maximum likelihood-derived univariate right tail parameter estimates for the VaR and expected shortfall measures.

Figure 10a. Quantile Plot of Right Tail - Log(FX Returns)

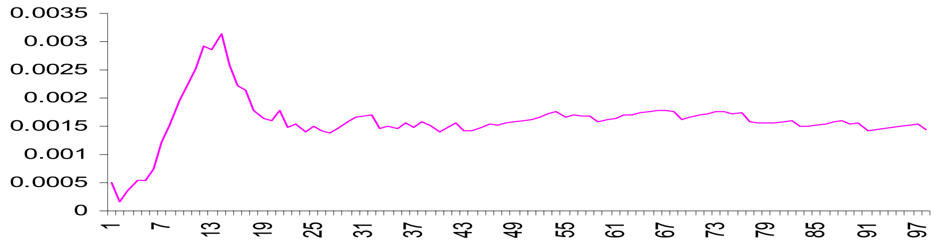


Figure 10b. Quantile Plot of Left Tail - Log(FX Returns)

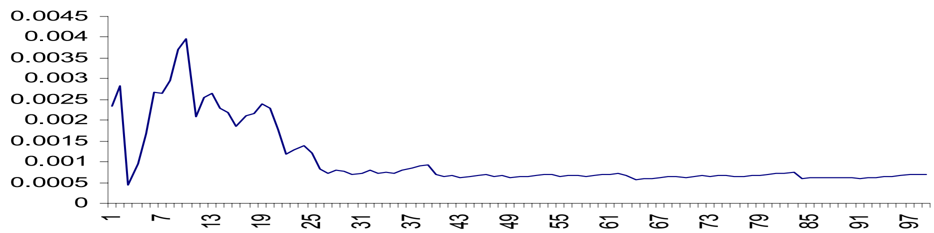


Figure 11a. Quantile Plot of Right Tail - Log(JSE Returns)

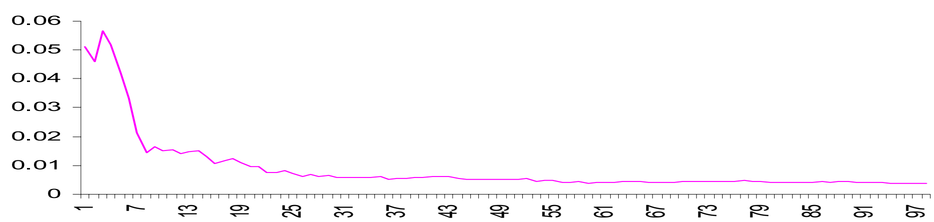
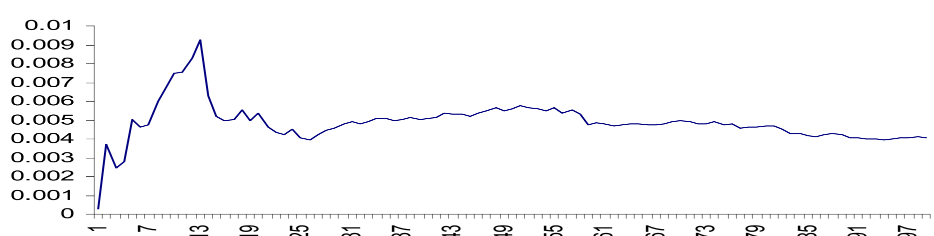


Figure 11b. Quantile Plot of Left Tail - Log(JSE Returns)



4.0 Maximum-Likelihood Estimation of Copulas

The parameters of the Gumbel-McFadden and the Hüsler-Reiss models are estimated using a two-stage procedure as follows.[31] First, the marginal tail estimation formulas (see equation [14]) are estimated by choosing the thresholds for each series to obtain:

$$\hat{F}_i(x) = 1 - \frac{N_{ui}}{n}(1 + \frac{\hat{\xi}_i}{\hat{\beta}_i}(x - u_i))^{-1/\hat{\xi}_i} \quad ,i = 1,...N.^{32} \qquad [23]$$

Then the extreme co-market dependence parameter, δ, is estimated using the specific copula function by maximum likelihood in the second stage using the density [5] in the case of the Gumbel-McFadden (GM) model and [7] in the case of the Hüsler-Reiss (HR) model. The copula function for the joint tail of F may be expressed as:

$$\hat{F}(x_1, x_2) = C_\delta^\kappa\left(\hat{F}_1(x_1), \hat{F}_2(x_2)\right) \quad ,\kappa = GM, HR. \qquad [24]$$

Let $\chi_{n,m}^+ = \max\{X_{n,1},...X_{n,m}\}$, θ be the $K \times 1$ vector of parameters to be estimated and $\hat{\theta}_{ML}$ denote the maximum-likelihood estimator. Using a sample of T observations, the log-likelihood is:

$$\ell(\theta) = \sum_{t=1}^{T} \ell_t(\theta) = \sum_{t=1}^{T} \ln c\left(\hat{F}_1(x_1^t),...,\hat{F}_n(x_n^t),...\hat{F}_N(x_N^t)\right) + \sum_{t=1}^{T}\sum_{n=1}^{N} \ln \hat{f}_n(x_n^t), \qquad [25]$$

where the density of the joint distribution F is given by:

$$f\left(x_1,...x_N\right) = c\left(F_1(x_1),...,F_N(x_N)\right)\prod_{n=1}^{N} f_n(x_n), \qquad [26]$$

and c is the density of the copula:

$$c(u_1,...,u_N) = \frac{\partial C(u_1,...u_N)}{\partial u_1 \cdots \partial u_n}. \qquad [27]$$

It may be shown that $\hat{\theta}_{ML}$ has the property of asymptotic normality[33] such that:

$$\sqrt{T}(\hat{\theta}_{ML} - \theta_0) \to N(0, \Im^{-1}(\theta_0)) \qquad [28]$$

with $\Im^{-1}(\theta_0)$ denoting the information matrix. Using the assumption of uniform margins, the log-likelihood may be expressed as:

$$\ell(\theta) = \sum_{t=1}^{T} \ln c(u_1^t,...,u_N^t).^{34} \qquad [29]$$

As earlier discussed, it is difficult to obtain very precise estimates of the marginal tail estimators. Thus, a vector of componentwise maxima is formed using 25, 30 and 40 trading day blocks to explore the

[31] See Joe and Hu (1996).
[32] $N = 2$ in this case.
[33] See Davidson and MacKinnon (1993).
[34] See Bouyé et al (2000).

sensitivity of the results to the length of maxima and minima.[35] Tables 3a to 3c present the MLE results of the bivariate (tail) dependence parameters, as well as the simple linear correlation parameter, $\hat{\rho}$, for comparison.[36] As shown by the four quadrants in Figure 12, $\chi_1^+ (\chi_1^-)$ denote the maxima (minima) FX returns and $\chi_2^+ (\chi_2^-)$ denote the maxima (minima) JSE returns. It is important to note that (χ_1^-, χ_2^+) and (χ_1^+, χ_2^-) represent a *flight to quality*, whereas (χ_1^+, χ_2^+) and (χ_1^-, χ_2^-) represent a foreign exchange-equity *co-boom* and *co-crash*, respectively.[37]

Figure 12. The Pairwise Dependence Structure for FX and JSE Extreme Returns

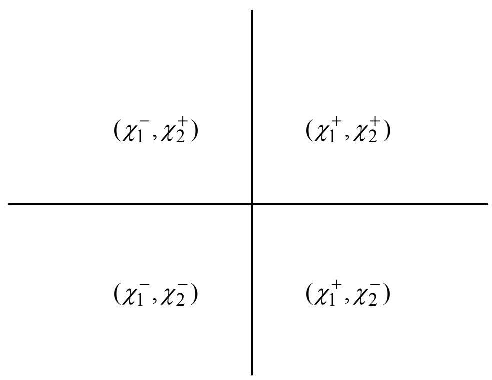


Table 3a. Pairwise Parameters for Extreme Dependence Between TTSE and JSE (Composite) Index (N=40)

Dependence Parameters	Simple (Pearson) Coefficient	Hüsler-Reiss Model	Gumbel-McFadden Model
	$\hat{\rho}$	$\hat{\lambda}$	$\hat{\delta}$
(χ_1^+, χ_2^+)	24%	61%	1.34
(χ_1^-, χ_2^+)	32%	100%	1.47
(χ_1^+, χ_2^-)	14%	40%	1.15
(χ_1^-, χ_2^-)	1%	0%	1.01

[35]The Durrlemann, Nikeghbali and Roncalli (2000) methodology may be used to choose between the Gumbel-McFadden and the Hüsler-Reiss copulas.
[36] The lower bound of the Pearson coefficient is constrained to equal 0.
[37] The Gumbel-McFadden parameter is interpreted similar to the *dependency measure* given in equation [2].

Table 3b. Pairwise Parameters for Extreme Dependence Between TTSE and JSE (Composite) Index (N=30)

Dependence Parameters	Simple (Pearson) Coefficient	Hüsler-Reiss Model	Gumbel-McFadden Model
	$\hat{\rho}$	$\hat{\lambda}$	$\hat{\delta}$
(χ_1^+, χ_2^+)	18%	61%	1.3
(χ_1^-, χ_2^+)	27%	92%	1.4
(χ_1^+, χ_2^-)	9%	51%	1.1
(χ_1^-, χ_2^-)	0%	0%	1.0

Table 3c. Pairwise Parameters for Extreme Dependence Between TTSE and JSE (Composite) Index (N=20)

Dependence Parameters	Simple (Pearson) Coefficient	Hüsler-Reiss Model	Gumbel-McFadden Model
	$\hat{\rho}$	$\hat{\lambda}$	$\hat{\delta}$
(χ_1^+, χ_2^+)	23%	71%	1.3
(χ_1^-, χ_2^+)	26%	61%	1.3
(χ_1^+, χ_2^-)	4%	41%	1.0
(χ_1^-, χ_2^-)	0%	0%	1.2

The results illustrated in Tables 3a to 3c clearly show that there exists strong evidence of extreme (tail) dependence between the foreign exchange and equity markets. Furthermore, consistent with the principles that underpin EVT, methods based on the normality assumption such as the Pearson correlation coefficient, *underestimate* extreme correlations. The results are also more or less consistent across dependency measures.

The results indicate the existence of the *flight to quality* phenomenon for all block sizes (ie for (χ_1^-, χ_2^+) and (χ_1^+, χ_2^-)). However, the magnitude of the parameters for each measure of dependence suggest that the likelihood of a flight to quality following a abnormally large appreciation (or a foreign exchange market crash) is greater than in the case of a stock market crash.

The results support the *cross-market arbitrage hypothesis* outlined in section 1 of this paper for all trading-day block sizes. That is, whereas all the dependence parameters are virtually equal to *zero* in the case of a co-crash for the foreign exchange and equity markets, in the event of a co-boom it is just as large (if not greater) than the case of a flight to quality. This support the hypothesis that the *rare* *occurrence* of a significant depreciation in the US/Jamaica Dollar exchange rate will simultaneously trigger a proportional rise in the JSE index, given the new demand for the cross-listed stocks.

5.0 Robustness Check

Tables 4a to 4c present the MLE results of the bivariate (tail) dependence parameters using the TTSE and the All Jamaica JSE index returns. The results indicate an *absence* of extreme co-dependence between the TTSE and All Jamaica JSE index returns in the cases of a co-boom and a flight to quality following a foreign exchange market crash for all three block sizes. Importantly, the non-existence of right-tail dependence between these series is interpreted as strong support for the existence of a significant cross-market arbitrage channel. The Hüsler-Reiss model shows evidence of 41 per cent dependence between a foreign exchange boom and a crash in the All Jamaica JSE index for the 30 and 40 trading day blocks. Finally, both extreme parameters reveal significant dependence in the case of a co-crash for all three block sizes.

Table 4a. Pairwise Parameters for Extreme Dependence Between TTSE and JSE (All Jamaica) Index (N=40)

Dependence Parameters	Simple (Pearson) Coefficient	Hüsler-Reiss Model	Gumbel-McFadden Model
	$\hat{\rho}$	$\hat{\lambda}$	$\hat{\delta}$
(χ_1^+, χ_2^+)	0%	0%	1.0
(χ_1^-, χ_2^+)	0%	0%	1.0
(χ_1^+, χ_2^-)	0%	0%	1.0
(χ_1^-, χ_2^-)	36%	71%	1.6

Table 4b. Pairwise Parameters for Extreme Dependence Between TTSE and JSE (All Jamaica) Index (N=30)

Dependence Parameters	Simple (Pearson) Coefficient	Hüsler-Reiss Model	Gumbel-McFadden Model
	$\hat{\rho}$	$\hat{\lambda}$	$\hat{\delta}$
(χ_1^+, χ_2^+)	0%	0%	1.0
(χ_1^-, χ_2^+)	0%	0%	1.0
(χ_1^+, χ_2^-)	0%	41%	1.0
(χ_1^-, χ_2^-)	30%	71%	1.4

Table 4c. Pairwise Parameters for Extreme Dependence Between TTSE and JSE (All Jamaica) Index (N=20)

Dependence Parameters	Simple (Pearson) Coefficient	Hüsler-Reiss Model	Gumbel-McFadden Model
	$\hat{\rho}$	$\hat{\lambda}$	$\hat{\delta}$
(χ_1^+, χ_2^+)	0%	0%	1.0
(χ_1^-, χ_2^+)	0%	0%	1.0
(χ_1^+, χ_2^-)	0%	41%	1.0
(χ_1^-, χ_2^-)	18%	61%	1.3

6.0 Concluding Remarks

The main aim of this paper was to answer the questions: (1) Does the data reflect the cross-market arbitrage channel resulting from the unique dependence between the Jamaican foreign exchange and equity markets that has been recently reported by financial analysts?; and (2) What are the implications of this cross-market arbitrage for risk managers? The paper uses EVT to account for the shortcomings of other types of correlation analysis.

The paper finds strong evidence of dependence between foreign exchange and equity market booms in Jamaica. However, the results show that this tail dependence is driven by the behaviour of cross-listed stocks during episodes of extreme foreign exchange rate depreciation. Furthermore, the probability of a co-crash is positive only when the cross-market stocks are excluded from the JSE composite index. Thus, risk managers can lower contagion risk from holding a portfolio of stocks and foreign exchange by decreasing the portfolio weights on stocks that are not cross-listed *relative* to cross-listed stocks. This constitutes strong evidence of stock market inefficiency. In order to exploit the benefits from this inefficiency, other Jamaican institutions that are listed on the JSE have recently began the process for cross-listing on the TTSE.

Financial system regulators have recently adopted the 1988 Basel Accord's simplified fixed-weight procedure for the calculation of capital adequacy requirements as standards of best practice. These standards, however, are focussed on credit risk. To improve prudential supervision, a market risk regulatory framework is presently being considered. However, regulators are faced with the dilemma of choosing between simple rules (such as imposing static limits on market risk) or establishing more complex regulation that are better able to accurately control excessive market risk. One important concern with regard to directing the use of complex risk management systems is the increasingly sophisticated risk modelling skills that would be required by regulatory and financial institutions' staff. However, this study highlights the importance of accounting for correlation and, hence, diversification, which is achieved by employing complex systems. Simple capital adequacy rules do not account for the dynamic dependence structure that exists among market factors. Thus, financial institutions are not rewarded from entering into offsetting transactions, which minimize their overall risk. This is inconsistent with the promotion of financial stability.

An important concern for regulators is that the use of empirical techniques to account for diversification may not be always accurate and may instead lead to inappropriate regulations. For example, Table 1 in this paper shows a low, negative correlation between foreign exchange and JSE index returns when the centres of their distributions are included in the computations, which implies offsetting risks. However, when likelihood of *extreme co-events* is computed by using only those observations in the "tails" of the distributions (see Tables 4a to 4c), the vulnerability of the portfolio to a co-crash is markedly evident. Also, consistent with the underpinnings of EVT analysis, the comparative results in this paper indicate that if risk managers use the simple correlation coefficient as an input in extreme risk analyses they would significantly underestimate VaR forecasts. Therefore, the challenge is for regulators to find the appropriate trade-off between the relative accuracy of including diversification benefits from complex regulations and a simple and manageable system to ensure conformity to regulations by all participants.

Finally, from the perspective of the JSE and TTSE indexes, the externalities from their *market segmentation* that are highlighted in this paper encourage the undesirable practice of speculation rather than promoting market depth based on institution fundamentals and exchange efficiency. In this regard, there have been recent initiatives by both exchanges to merge under the umbrella of a single regional stock exchange. This is expected to eliminate arbitrage channels and contribute to the equity market depth, liquidity and resiliency.

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Appendix A.

Table IA. Companies Listed on the JSE Index as at June 2003
Bank of Nova Scotia (Jamaica)
Berger Paints (Jamaica)
Cable & Wireless (Jamaica)
Capital and credit merchant bank
Caribbean Cement
Carreras Group
Ciboney Group
CMP Industries
Courts (Jamaica)
Dehring, Bunting & Golding
Desnoes & Geddes
Dyoll Group
First Caribbean International Bank
First Caribbean International Bank (JA)
First Life Insurance
Gleaner Company
Goodyear (Jamaica)
Grace, Kennedy & Co.
Guardian Holdings Limited
Hardware & Lumber
Island Life Insurance
Jamaica Broilers Group
JMMB Ltd.
Jamaica Producers Group
Kingston Wharves
Lascelles, de Mercado
Life of Jamaica
Montego Freeport
Mobay Ice Company
National Commercial Bank Jamaica
Palace Amusement
Pan Caribbean Inv. (Trafalgar Dev)
Pan Jam Investments
Pegasus Hotel
Radio Jamaica
RBTT Financial Holdings Limited
Salada Foods
Seprod
Trinidad Cement Limited
West Indies Pulp & Paper

Table IIA. Companies Listed on the All Jamaica JSE Index as at June 2003
Bank of Nova Scotia (Jamaica)
Berger Paints (Jamaica)
Cable & Wireless (Jamaica)
Caribbean Cement
Carreras Group
Courts (Jamaica)
Desnoes & Geddes
First Life Insurance
Gleaner Company
Grace, Kennedy & Co.
Jamaica Broilers Group
Jamaica Producers Group
Lascelles, de Mercado
National Commercial Bank Jamaica
Radio Jamaica

Table IIIA. Stocks Cross-Listed on the JSE and TTSE Index as at June 2003
First Caribbean International Bank
Grace, Kennedy & Co.
Guardian Holdings Limited
JMMB Ltd.
RBTT Financial Holdings Limited
Trinidad Cement Limited

Appendix B.

Maximum-Likelihood Estimation of Univariate GPD Parameters

As a more precise alternative to the Hill plot, the maximum likelihood procedure can be used to estimate the GPD parameters, $\hat{\xi}$ and $\hat{\beta}$. The log-likelihood function for the GPD is given as:

$$\ell(\xi,\beta) = \begin{cases} -N\ln\beta - \left(\frac{1}{\xi}+1\right)\sum_{n=1}^{N}\ln\left(1+\frac{\xi}{\beta}x_i\right), & \text{if } \xi \neq 0 \\ -N\ln\beta - \frac{1}{\beta}\sum_{n=1}^{N}x_i, & \text{if } \xi = 0. \end{cases} \quad [22]$$

For illustration, the results for the foreign exchange and JSE positive returns are presented in Table 3. The table also includes the computed VaR, from [15], and expected shortfall, from [17], measures for $p = 0.95$.

Table 3. MLE Results for Positive FX and JSE Returns

Parameter	**Estimate**	**Bootstrap Confidence Interval**	**Mean Square Error**
FX Positive Returns:			
$\hat{\xi}$	0.035	(0.030, 0.043)	0.00001
$\hat{\beta}$	0.271	(0.225, 0.371)	0.00060
$\hat{x}_{0.95}$	0.163	-	-
$\hat{S}_{0.95}$	0.449	-	-
JSE Positive Returns:			
$\hat{\xi}$	0.135	(0.099, 0.179)	0.00039
$\hat{\beta}$	0.747	(0.604, 0.864)	0.00441
$\hat{x}_{0.95}$	0.071	-	-
$\hat{S}_{0.95}$	0.953	-	-