An Assessment of Volatility Transmission in the Jamaican Financial System

Percival Hurditt
Financial Stability Department
Research and Economic Programming Division
Bank of Jamaica

Abstract

This paper applies the GARCH-BEKK procedure to the returns from the Jamaican bond, foreign exchange and stock markets in order to estimate the magnitude of the common market and cross-market volatility transmission. The behaviour of these spillover effects over a specified period is then assessed. In particular, the paper employs a simple VAR procedure that uses the variance series of the three market returns derived from the GARCH-BEKK model as the endogenous variables. The results of the model suggest that within the Jamaican financial system, there are generally high levels of common market volatility transmission, relative to cross-market volatility transmission. Of the three markets, the foreign exchange market exhibits the most pronounced common market volatility transmission, followed by the stock market. Strong common market transmission in these two markets, relative to that of the bond market reflects the uncertainty momentum that often characterises these risky markets. The strongest cross-market effects occur from the bond market to the foreign exchange and stock markets. Findings of the paper suggest a negligible impact of GOJ and BOJ bond maturities on volatility transmission within and across markets. This is interpreted as evidence of the successful management of liquidity through open market operations.

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1. Introduction

Market prices are generally influenced by the portfolio decisions of investors who actively participate in more than one financial market. In turn, these decisions are usually influenced by a continuous flow of information that often results in market price volatility spillovers within and across markets[1]. Market efficiency proponents generally attribute the spillover effects to inefficiencies in market structures, particularly in the dissemination of relevant information to market participants. These spillovers could reflect a failure of market efficiency as it should not be possible to predict returns or volatility in one market using lagged information. However, if news about fundamentals were serially correlated, then the existence of spillovers need not imply a failure of market efficiency[2].

Close examination of the nature of volatility transmission is important in aiding the effectiveness of monetary policy and in addressing financial stability issues. With regard to monetary policy, it is critical to understand the manner in which shocks are propagated across markets in order to determine the persistence of these innovations and the magnitude of their effects over time.[3] The extent to which volatility is transmitted across markets could result in a large shock in one market destabilizing another market. The ability of policy-makers to gauge the depth and duration of the impact of cross-market and common market shocks could aid the implementation of timely and effective monetary policy. As a financial stability concern, it is also extremely important to understand the various market price interrelationships. The complexity of these interrelationships represents a potential source of systemic financial instability. To this extent, a comprehension of the intricate market return volatility linkages facilitates the implementation of effective mechanisms that allow or encourage entities to hedge against the market risks emanating from shocks that persist within a financial market and those that are propagated across markets.

[1] In this paper, the term "volatility spillover" represents the common-market case in which historical volatility in a particular market impacts the current volatility in the same market, as well as the cross-market case in which the historical volatility in one market impacts the current volatility in another market.
[2] See Ebrahim (2000).
[3] See Ebrahim (2000).

A useful explanation of the source of volatility spillovers is offered by modern portfolio theory. Beginning with Markowitz (1952), this theory has established the importance of financial asset risk and return in the determination of investor demand for a financial asset. In a basic portfolio model, an investor finds an optimal balance between portfolio risk and return by maximising an asset portfolio return for a given level of risk. Within this framework, the portfolio return reflects the weighted average of the returns from the various assets included in the portfolio, while total portfolio risk is determined by the volatility of the return on each asset group and the joint volatility between the return on all the paired combinations of assets in the portfolio.

Fleming, Kirby and Ostdiek (1996) provide a useful theoretical explanation for price volatility behaviour. Using a simple model of speculative trading, they employ mean-variance portfolio optimization, as proposed by Markowitz (1952), to derive a theoretical relationship between the demand for asset "futures" and the risk and return of the underlying assets. This relationship provides an implied link between the demand for financial assets and the market return volatilities. In a dynamic setting, the asset returns volatilities impact the demand for the asset, which could cause bouts of common market and cross-market volatilities in subsequent periods. Common market volatility arises from investor uncertainty induced from the initial shock event to the return of an asset. In explaining the case of cross-market volatility spillovers, Fleming, Kirby and Ostdiek (1996) assert that, as a portfolio manager considers the correlation between different market returns, he will take a position in one market in order to hedge his speculative position in another. In addition to the hedging channel, the model also indicates that cross-market volatility spillovers may generally occur where an information event that alters the expectation about returns in one market will influence demand and trading in another market.

This paper will apply the multivariate form of the GARCH procedure[4] to the returns from the Jamaican bond, foreign exchange and stock markets as an empirical complement to the theoretical description of asset returns volatilities proposed by Fleming, Kirby and

[4] This procedure was established by Baba, Engle, Kraft, and Kroner (BEKK) in 1991.

Ostdiek (1996). This empirical model will be used to estimate the coefficients reflecting the extent of common market and cross-market volatility spillovers. Importantly, the influence of changes in market liquidity, in terms of bond maturities may need to be explicitly accounted for when computing volatility spillovers. Accordingly, the GARCH-BEKK procedure is carried out *without the inclusion of liquidity effects* in the model and *with the inclusion of liquidity effects* in the model, so as to gauge the impact of Jamaica Dollar liquidity on the asset return volatility linkages.[5] In addition to the model estimations of volatility spillovers, the paper will seek to utilize the modelled variance series as inputs in a simple vector autoregressive (VAR) model to produce ten-day volatility impulse responses. This application details the extent to which the variance of the asset return in a particular market is influenced by the lagged variances of the returns in the same market and the other two markets.

The remainder of the paper will be organized in the following manner. Section two of the paper takes an in-depth look at the proposed volatility-trading model that provides some intuition for volatility connections. Section three provides a brief literature review on some of the applications of autoregressive and the generalized auto regressive time series models. Section four outlines the specification of the multivariate BEKK model that will be employed and data is described in section five. The sixth section looks at the estimation results and discusses the findings. The final section presents the conclusion and policy recommendations.

2. Theoretical Motivation

Following on work done by Tauchen and Pitts (1983), Fleming, Kirby and Ostdiek (1996), utilized the mean-variance optimisation framework to construct a trading model for financial asset futures. The model assumes that the economy contains a large number of active speculators, who trade with one another because they differ in their expectation about the future and in their need to transfer risks through market transactions. At the start of a trading round, all the financial markets are in equilibrium. This paper adapts the

[5] Although many volatility models concentrate on utilizing historical uncertainty measures to estimate conditional volatility, studies such as that done by Hamilton and Lin (1996) show that certain exogenous variables affect volatility. In this study the effect of domestic dollar liquidity is considered.

framework of Fleming, Kirby and Ostdiek (1996) but modifies the assumptions to reflect the absence of a futures market in Jamaica.

When new information arrives, traders revise their demand for a particular financial instrument and the information event generates a round of trading that continues until the market price has reached a new equilibrium. Formally, let S_t^a be the underlying asset price at time t and $E\left[S_T^a\right]$ be the expected price of the asset a at time t that the investor expects to receive when the asset is sold at time T. A speculator, who takes a long position at time t, expects to earn a profit of $\pi_{t,T}^a$ at time T, where the expectations are conditioned on all available information. The expected profit for the speculator, given his information set, I_t, is given as:

$$E\left[\pi_{t,T}^a / I_t\right] \equiv E\left[S_T^a\right] - S_t^a \qquad (1)$$

Given the above conditions, the portfolio mean-variance optimization (MVO) theory may be used to derive the demand for the asset. The theory assumes that the trader maximizes his expected profit subject to a variance constraint. The standard results derived from this theoretical framework suggest that:

$$Q^a = \frac{E\left[\pi_{t,T}^a\right]}{2\alpha\sigma^2} \qquad (2)$$

where Q^a is the quantity demanded for asset a, α is the trader's risk aversion coefficient and σ^2 is the variance of the expected return. The impact on the asset variance in equation (2) highlights a channel through which asset return volatility in one period, by influencing demand and demand expectations of the speculative trader, may impact the future volatility of the asset.

The volatility-trading model may be generalized to allow speculators to trade in more than one asset market, market b and market s, for example. In this case, the demand function exhibits cross-market dependencies. In the case where the two assets are traded, let β denote the slope coefficient in the linear regression of the expected profit, $\pi^s_{t,T}$, for market s, on the expected profit, $\pi^b_{t,T}$, for market b. Further, let $\sigma^2_{s/b}$ represent the variance of the regression error. Similarly, define β_b and $\sigma^2_{b/s}$ as denoting the slope coefficient and the variance for the regression of the expected value of $\pi^b_{t,T}$ on the expected value of $\pi^s_{t,T}$. For these conditions, the MVO theory derives the following demand functions:

$$Q_s = \frac{E[\pi^s_{t,T}]}{2\alpha\sigma^2_{s/b}} - \frac{E[\pi^b_{t,T}]}{2\alpha\sigma^2_{b/s}}\beta_b \tag{3}$$

and

$$Q_b = \frac{E[\pi^b_{t,T}]}{2\alpha\sigma^2_{b/s}} - \frac{E[\pi^s_{t,T}]}{2\alpha\sigma^2_{s/b}}\beta_s \tag{4}$$

The demand functions, Q_s and Q_b denote the number of assets s and b, respectively, which are demanded by the speculative trader who is betting on the future movements in the underlying asset prices in order to maximize his overall portfolio returns. The model shows that in the two-market case, the trader's demand for asset s is a function of $\sigma^2_{s/b}$, so the size of his position in this asset depends on the cross variance between the expected profits on asset s and asset b. This cross-variance term justifies a volatility impact from one asset, impacting the demand and thus the future volatility of the other asset in the portfolio.

The demand functions, depicted by equations (3) and (4), suggest that the trader's demand for an asset, in the two market case, is a function of the expected profits and the cross variances $\sigma^2_{s/b}$ and $\sigma^2_{b/s}$. Fleming, Kirby and Ostdiek (1996) explain that the cross variances may result from a hedging channel. In explaining this channel they note that if the expected profit for one financial asset is zero, the demand could still be non-zero. For example, the model shows that even if the expected profit for asset b is zero, once the investor believes that there is a negative correlation between the two markets, there will still be a positive demand for asset b equal to $-\beta_S Q_S$ [6]. This demand occurs because there are hedging benefits in holding the asset to reduce the overall portfolio risk as long as the two assets have a negative correlation. With the hedge in place, the risk of a long position in the asset is $\sigma^2_{s/b}$. This risk is generally less than the risk of an unhedged asset S, so the trader's demand for asset s is increased by the availability of asset b. Where there is news pertaining to the market for asset s, due to hedging, it will lead to an adjustment in the demand for asset s and also the demand for the substitute asset b. Changes in the spot prices of the assets due to hedging would be immediately reflected in the expected future spot price through the no-arbitrage pricing relation. This is the hedging channel through which information in one market may spill over into another market. Equations (3) and (4) also suggest also provide a broader interpretation of cross-market volatility spillovers. The fact that the demand for one asset is also dependent on the variance of all the assets in the portfolio, the model implies that an information event that alters the expectation about returns in one market will influence demand and trading in another market. This is a general explanation for cross-market volatility spillovers.

3. Review of Literature on Asset Price Volatility Spillovers

It has been widely observed that financial time series tend to exhibit volatility clusters. Mandelbrot (1963) and Fama (1965) both reported evidence that large changes in the price of an asset are often followed by other large changes and, similarly, small changes are often followed by small changes. A number of other studies, including Baillie et al (1996), Chou (1998) and Schwert (1989) report similar findings. In order to capture these volatilities, the autoregressive conditional heteroskedasticity model (ARCH) of Engle (1982) and the generalized extension of this model (GARCH) by Bollerslev (1986) have been used extensively in financial analyses. In a simple case of the application of the ARCH methodology on a single return series, the variance in the return series is modelled as a function of past variances and past errors that are derived from the regression of the mean return series on lagged versions of itself. Maximum Likelihood Estimations are

[6] Negative correlation of returns gives a negative β_s.

then used to compute the coefficients of the model. It has been found (Chong, et al, 1999 and Gokcan, 2000), that ARCH and GARCH models provide good in-sample parameter estimates and, when the appropriate volatility measure is used, reliable out-of-sample volatility forecasts. Among these models, there has been, particularly, much support for the GARCH (1,1) model[7]. In an extensive application of 330 different volatility models to daily US/ DM exchange rates and the daily stock price of IBM, Hansen and Lunde (2001) conducted out-of-sample comparisons that revealed that none of these models provide a significantly better forecast than the GARCH (1,1) model.

A number of univariate GARCH models have been applied to measuring the extent of volatility spillovers from one financial asset return series to another. In one application, Kim and Langrin (1998) used an asymmetric version of the GARCH methodology to model the conditional mean and variance equations for stock price returns in Jamaica and Trinidad and Tobago with spillover effects from the US stock market[8]. This study revealed the existence of spillover effects from a major US stock market to the markets in Jamaica and Trinidad and Tobago. In a more recent application of the univariate GARCH model, Bala and Premaratne (2002) applied the model to daily stock market returns to reveal some degree of stock market volatility spillovers between the stock market in Singapore and the markets in US, Japan and Hong Kong. In another application, Song and Small (2002) used daily returns series to reveal that there was significant spillover effects from major US stock market return indicators to daily individual stock market returns. In another study involving a combination of developed financial markets Kaltenhaeuser (2003) found evidence of stock market volatility spillovers between the markets in the euro area, US, and Japan. These studies indicate the widespread occurrence of financial market volatility spillovers.

Although the univariate GARCH approach has a history of success in capturing the effects of volatility spillovers, it has shortcomings in two aspects. Firstly, the information contained in variance-covariance matrix of residuals derived from the univariate GARCH

[7] See Hansen (2001).

[8] In this study, the authors accounted for the leverage effects that usually cause stock prices to respond more intensively to negative news, relative to positive news.

framework cannot be utilized in estimating the effects of volatility spillovers. Secondly, these models involve problems in estimating efficiency and unbiasedness. In order to counter these problems the multivariate approach serves as a natural extension of the univariate GARCH approach, which explicitly accounts for these shortcomings.

A number of multivariate GARCH models have been proposed in the literature. One of the earliest rigorous attempts in this category was the VECH model of Bollerslev, Engle, and Wooldridge (1988). This approach extended the basic model of Engle and Bollerslev by using the simultaneous equation form of the original model. The VECH model proved to be a cumbersome approach, as a large number of coefficients had to be estimated, thus utilizing relatively small degrees of freedom in the estimation process. To curb this estimation problem, Bollerslev (1990) introduced the Constant Conditional Correlation (CCC) model. This model simplified the estimation of the multivariate GARCH coefficients by imposing restrictions on the variance-covariance matrix that was derived from the system of simultaneous equations. Although the CCC model was a useful improvement over the VECH model of time variant volatilities in financial time series, this model had apparent drawbacks. Firstly, the major assumption of constant correlations between the different variables in the system of equations was thought to be unrealistic[9]. Furthermore, the model did not ensure that the estimated variance-covariance matrix was positive definite. The positive definiteness of the variance-covariance matrix was a necessary mathematical condition to guarantee the solvability of the system of equations and hence the estimation of the model coefficients.

In order to avoid the unrealistic assumptions on the variance-covariance matrix and to circumvent the problem of non-positive definiteness of the variance-covariance matrix, Engle and Kroner (1995) proposed the BEKK model -named after Baba, Engle, Kraft, and Kroner (1991). This model uses a quadratic form of the parameterization of the original system of equations to ensure the positive definiteness of the variance-covariance matrix without significantly changing the information content of the system of

[9] For example, Login and Solink (1995), and Karolyi and Stulz (1996) found evidences of time–varying conditional correlations between international equity markets.

equations[10]. In one application of this model, Ebrahim (2000) used a trivariate case to study price and volatility spillovers between the foreign exchange and associated money markets for three different countries, relative to the US.

4. Empirical Model

The trivariate representation of the BEKK model is adopted in this paper to examine the volatilities and pair-wise volatility linkages between the Jamaican stock, bond and foreign exchange markets. In this application, the following BEKK form will be used to model the asset returns and returns volatility of the bond market, foreign exchange market and stock market, labelled as assets 1, 2 and 3, respectively.

The mean equation:

$$R_t = \alpha + \beta' R_{t-1} + \varepsilon_t \tag{5}$$

where:

$$R_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \\ r_{3,t} \end{bmatrix} \; ; \; \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \; ; \; \beta = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \; ; \; \varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} .$$

The vector R_t represents the returns for the bond market, the foreign exchange market and the stock market, respectively, at time t. The α_t vector and the β_t matrix represent the coefficients in the mean equation and the ε_t vector represents the errors in the mean equation. In this formulation, equation (5) represents a vector autoregressive model with a single lag in the endogenous variables.

The associated variance-covariance equation is represented by:

$$\Sigma t = C'C + A'\varepsilon t - 1\varepsilon' t - 1A + B'\Sigma t - 1B \tag{6}$$

where:

[10] There are eleven parameters to be estimated in the bivariate form of this model.

$$\Sigma_t = \begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} & \sigma_{13,t} \\ \sigma_{21,t} & \sigma_{22,t} & \sigma_{23,t} \\ \sigma_{31,t} & \sigma_{32,t} & \sigma_{33,t} \end{bmatrix} \; ; \; C = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \; ; \; A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \; ; \; B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}.$$

The variance-covariance matrix (6) yields the following equations:[11]

$$\begin{aligned} \sigma_{11,t} = c_{11}^2 + c_{21}^2 + c_{31}^2 + a_{11}&\left(a_{11}\varepsilon_{11,t-1} + a_{21}\varepsilon_{21,t-1} + a_{31}\varepsilon_{31,t-1}\right) + \\ a_{21}&\left(a_{11}\varepsilon_{12,t-1} + a_{21}\varepsilon_{22,t-1} + a_{31}\varepsilon_{32,t-1}\right) + \\ a_{31}&\left(a_{11}\varepsilon_{13,t-1} + a_{21}\varepsilon_{23,t-1} + a_{31}\varepsilon_{33,t-1}\right) + \\ b_{11}&\left(b_{11}\sigma_{11,t-1} + b_{21}\sigma_{21,t-1} + b_{31}\sigma_{31,t-1}\right) + \\ b_{21}&\left(b_{11}\sigma_{12,t-1} + b_{21}\sigma_{22,t-1} + b_{31}\sigma_{32,t-1}\right) + \\ b_{31}&\left(b_{11}\sigma_{13,t-1} + b_{21}\sigma_{23,t-1} + b_{31}\sigma_{33,t-1}\right) \end{aligned} \tag{7}$$

$$\begin{aligned} \sigma_{22,t} = c_{22}^2 + c_{32}^2 + a_{12}&\left(a_{12}\varepsilon_{11,t-1} + a_{22}\varepsilon_{21,t-1} + a_{32}\varepsilon_{31,t-1}\right) + \\ a_{22}&\left(a_{12}\varepsilon_{12,t-1} + a_{22}\varepsilon_{22,t-1} + a_{32}\varepsilon_{32,t-1}\right) + \\ a_{32}&\left(a_{12}\varepsilon_{13,t-1} + a_{22}\varepsilon_{23,t-1} + a_{32}\varepsilon_{33,t-1}\right) + \\ b_{12}&\left(b_{12}\sigma_{11,t-1} + b_{22}\sigma_{21,t-1} + b_{32}\sigma_{31,t-1}\right) + \\ b_{22}&\left(b_{12}\sigma_{12,t-1} + b_{22}\sigma_{22,t-1} + b_{32}\sigma_{32,t-1}\right) + \\ b_{32}&\left(b_{12}\sigma_{13,t-1} + b_{22}\sigma_{23,t-1} + b_{32}\sigma_{33,t-1}\right) \end{aligned} \tag{8}$$

$$\begin{aligned} \sigma_{33,t} = c_{33}^2 + a_{13}&\left(a_{13}\varepsilon_{11,t-1} + a_{23}\varepsilon_{21,t-1} + a_{33}\varepsilon_{31,t-1}\right) + \\ a_{23}&\left(a_{13}\varepsilon_{12,t-1} + a_{23}\varepsilon_{22,t-1} + a_{33}\varepsilon_{32,t-1}\right) + \\ a_{33}&\left(a_{13}\varepsilon_{13,t-1} + a_{23}\varepsilon_{23,t-1} + a_{33}\varepsilon_{33,t-1}\right) + \\ b_{13}&\left(b_{13}\sigma_{11,t-1} + b_{23}\sigma_{21,t-1} + b_{33}\sigma_{31,t-1}\right) + \\ b_{23}&\left(b_{13}\sigma_{12,t-1} + b_{23}\sigma_{22,t-1} + b_{33}\sigma_{32,t-1}\right) + \\ b_{33}&\left(b_{13}\sigma_{13,t-1} + b_{23}\sigma_{23,t-1} + b_{33}\sigma_{33,t-1}\right) \end{aligned} \tag{9}$$

[11] Note here that $\varepsilon_{ii} = \varepsilon_i^2$ and $\varepsilon_{ij} = \varepsilon_i \times \varepsilon_j$.

Equation (6) represents the BEKK formulation of the trivariate GARCH procedure,[12] which reflects the quadratic form of the multivariate GARCH. In the equation, Σ_t represents the variance-covariance matrix of the GARCH methodology. Within this framework, the matrix C is a lower triangular matrix that is used to derive the constants for the variance equation. The ARCH and GARCH coefficients are derived from the A and B matrices.

The variance equations shown in equations (7) to (9) provide a full description of the factors that influence the asset return volatility within the context of the trivariate BEKK framework. These equations suggest that the asset variances are dependent on constants, the lag of squared residual terms, the products of lags of cross residual terms, the lags of variances and the lags of co-variances. In observing volatility spillovers, it is necessary to measure the impact of the lagged squared residuals $\varepsilon_{11,t-1}$, $\varepsilon_{22,t-1}$ and $\varepsilon_{33,t-1}$ or the effect of the lagged variances $\sigma_{11,t-1}$, $\sigma_{22,t-1}$ and $\sigma_{33,t-1}$ on the variances on the asset return volatilities/variances $\sigma_{11,t}$, $\sigma_{22,t}$ and $\sigma_{33,t}$.[13]

In comparing the two possible sets of spillover effects, Edy (2002) posits that the effect of the lagged variances on the present variances is delayed. He notes that shocks in asset return variances would first take effect through the squared residuals and the impact of the lagged variance on the present variance is reflective of a second round effect occurring within the variance equation. Specifically, it is indicated that a change in $\sigma_{ii,t-1}$ is partially dependent on a shock in period $t-2$, where $\varepsilon_{ii,t-1}$ is the indicator of a volatility spillover. For example, the square of the coefficient, a_{21}, measures the impact of the second market's (foreign exchange market) return on that of the first (bond) market. Similarly, the square of the coefficient, a_{12}, will be relevant for measuring the effect of the first (bond) market on the volatility of the second (foreign exchange) market in the model.

[12] For purposes of illustration, the covariance equations were omitted. See appendix for the full model consisting of three variance equations and six covariance equations.
[13] At this point the model also allows us to measure the effect of lagged variances on the present variance.

5. Data

The study utilises the daily frequency of the main Jamaica Stock Exchange (JSE) Index, the 30-day private repurchase agreement rates and the weighted average selling exchange rate to compute the continuously compounded market returns as follows:

$$r_t = 100 \times \ln\left[\frac{p_t}{p_{t-1}}\right] \tag{10}$$

In this case, r_t is the rate of return, 'ln' represents the natural logarithm and p_t is the market price. There are 518 data points ranging from 07 February 2002 to 17 November 2003. This data range involves periods of significant market volatility in all three markets, as well as periods of relative calm (see Appendix for plot of data series). From these series, the nominal capital pre-tax gains are computed to generate a representative asset return from each of the three major financial markets: the stock market, the bond market and the foreign exchange market[14].

For purposes of the study, the 30-day private interest rate on bonds traded in Jamaica's money market is used. The money market encompasses a wide cross-section of public and private securities of various maturities. This rate was selected on the basis of continuity in the series and its consistency in mirroring the rates on public money and bond market securities, bank lending rates and other private rates. Using the equivalent yield transformation, the selected interest rates are converted to a daily series. At the 5 per cent level of significance the augmented Dickey Fuller rejects the null hypothesis of a unit root in the data generating process of the bond market return series (see Appendix). Additionally, the Jarque – Bera statistic rejects the normality hypothesis[15]. As depicted in Figure A1, the relatively high coefficient of skewness (1.308) and the kurtosis of 4.584, reflects the high concentration of bond market returns in the lower end of the distribution. The low dispersion of the bond market return, relative to the other market returns is reflected in the relatively small standard deviation (0.017). While, the high kurtosis is typical of financial market returns, the relatively low dispersion is a fair reflection of

[14] See data series in appendix.
[15] A normally distributed serried reflects a skewness coefficient of zero (0) and a kurtosis of three (3).

what usually obtains in the fixed income market. The Q – statistics corresponding to the series residuals reveal the existence of ARCH effects.

The foreign exchange market return is computed as the logged difference of the daily weighted average selling exchange rate series that is recorded from foreign exchange market operations in the Jamaican financial system. These returns provide an average representation of the returns that can be derived from a number of segmented foreign exchange markets across the Jamaican economy. Similar to the case of the bond market return, there is no evidence of non-stationarity at the 5.0 per cent level of significance for foreign exchange market returns. The null hypothesis of normality in the distribution is also rejected due to a significant kurtosis of 59.2 and the negative skewness in the data reflecting the impact of a small number of negative returns in the left tail of the return distribution (see Figure A2). The high kurtosis is again reflecting the bunching of returns in the upper half of the distribution that is reflective of the depreciation bias in the domestic foreign exchange market. The foreign exchange return series also exhibits significant Arch effects.

The main JSE Index is a market value weighted index that includes all the entities listed on the JSE, and is therefore the broadest representation of prices of stocks traded on the equity market. The logged difference of this index gives the stock market returns. The stock market return series also rejects the unit root hypothesis at the 5.0 per cent level of significance. The positive skewness and fairly high standard deviation in the data indicates the existence of a few cases of very high stock market return. The data exhibits extremely high kurtosis. Similar to the other two markets, tests of the residual reveal significant ARCH effects.

In order to isolate the volatility transmission impact in each market due to uncertainty, it is necessary to account for the exogenous liquidity effects in the model. That is, contemporaneous and lagged volatility linkages within and between markets might be caused by significant changes in market liquidity. Thus, it is important to distinguish between these sources of volatility spillovers in order to determine the significance of the respective transmission channels. Liquidity in the model is represented by the sum of the total maturities of Government of Jamaica (GOJ) bonds and Bank of Jamaica (BOJ) Open Market Operation instruments, measured in Jamaica Dollars. This liquidity variable is measured according to daily logarithm values.

6. Empirical Results

Tables 1 and 2 give the GARCH estimation results for the model with and without the inclusion of the liquidity variable, respectively. A comparison of the results of the two models provides useful information on the influence of liquidity effects on the volatility linkages between markets. In the case where liquidity is explicitly accounted for in the model, the results reflect volatilities that would prevail in the absence of maturities. In the case where the liquidity effects are absent from the model, the volatility coefficients reflect the impact of Government and Central Bank bond maturities.

Table 1
GARCH results accounting for liquidity effects

Exogenous coefficients	Coefficient value	P-value
A-Matrix		
a11	0.047500	0.0000
a21	0.000209	0.0000
a31	-0.000040	0.0000
a12	-0.019212	0.0000
a22	0.051649	0.0000
a32	-0.000306	0.0000
a13	0.049754	0.0000
a23	-0.003927	0.0000
a33	0.050806	0.0000
B-Matrix		
b11	0.056088	0.1761
b21	0.005271	0.0000
b31	-0.006141	0.0437
b12	6.099788	0.0000
b22	0.093083	0.0000
b32	-0.103554	0.0000
b13	-6.597950	0.0000
b23	-0.106508	0.0000
b33	0.190792	0.0001

Table 2
GARCH results not accounting for liquidity effects

Exogenous coefficients	Coefficient value	P-value
A-Matrix		
a11	0.047495	0.0000
a21	0.000210	0.0000
a31	-0.000040	0.0000
a12	-0.019258	0.0000
a22	0.051652	0.0000
a32	-0.000307	0.0000
a13	0.049911	0.0000
a23	-0.003940	0.0000
a33	0.050809	0.0000
B-Matrix		
b11	0.057167	0.1512
b21	0.005297	0.0000
b31	-0.006351	0.0598
b12	6.106732	0.0000
b22	0.093223	0.0000
b32	-0.103882	0.0000
b13	-6.571240	0.0000
b23	-0.106705	0.0000
b33	0.191231	0.0006

The model coefficients depicted in Tables 1 and 2, when combined as specified by equations (7) to (9), reveal the impact of the squared residuals on the different asset variances. The new coefficients derived from these combinations suggest the extent of volatility linkages. For example, a_{21}^2 represents the extent of volatility spillovers from the foreign exchange market to the bond market and a_{12}^2 reflects the extent of volatility spillovers from the bond market to the foreign exchange market. Similarly, a_{13}^2 measures the extent to which there is a volatility spillover from the bond market to the stock market and a_{31}^2 represents the extent of volatility spillovers from the stock market to the bond market. With regard to the relationship between the foreign exchange and the stock market, a_{23}^2 represents the extent to which there are volatility spillovers from the foreign exchange market to the stock market, while a_{32}^2 represents the extent to which there is volatility spillovers from the stock market to the foreign exchange market. For purposes of completeness, the common market volatility effects may also be assessed. In this case, the coefficients a_{11}^2, a_{22}^2 and a_{33}^2 represent the effect of the squared bond market residual at time $t-1$ on the bond market volatility at time t, the effect of the squared foreign exchange market residual at time $t-1$ on the foreign exchange market volatility at time t and the effect of the squared stock market residual at time $t-1$ on the stock market volatility at time t.

Table 3 shows the common market volatility spillover results of the model for the case where liquidity effects are explicitly accounted for in the model[16]. The results reveal that, abstracting from the influence of new liquidity to the system, the impact on all three market returns volatilities arising from a shock from the previous day is statistically significant. This indicates the existence of strong serial correlation in bond market returns, foreign exchange market returns and stock market returns. The results suggest, however, that the common-market impact of bond volatility is the weakest of the three markets. This followed by the stock market common-market impact, while the strongest common-market impact occurs in the foreign exchange market. The relative strength of the common market transmissions reflects the relative volatility momentum exhibited by these markets.

The results generally reflect weaker cross-market volatility spillover effects than for the common market cases. The strongest spillover impact occurs from the bond market to the stock market. The asymmetry in the flow of volatilities between these two domestic markets is clearly reflected by the smallest spillover impact from the stock market to the bond market. The significant asymmetry between the strength of the volatility spillovers between these two markets might reflect the rapid adjustments in the stock prices of highly capitalised financial entities whose portfolios are heavily exposed to interest rate

[16] Engle and Sheppard (2001) note the difficulties in interpreting the absoluteness of the volatility spillover coefficients derived from the BEKK model. However, the relative magnitude of the spillover coefficients is useful in comparing common market and cross-market spillover effects.

risks. Further, the relatively weak volatility spillover from the stock market to the bond market might be due to the relative size differential in the markets. In terms of investment flows, the Jamaican bond market is far larger than the domestic stock market. Consequently, the sequential volatility channel from the stock market to the bond market is weak.

Table 3
Factors that influence market volatilities
not accounting for liquidity in the model

Empirical volatility transmission channels			
Common market volatility			
	Bond variance	**Foreign exchange variance**	**Stock variance**
Bond	0.0022562866	-	-
Foreign exchange	-	0.0026676487	-
Stock	-	-	0.0025812333
Cross market volatility (effect of squared residuals on variances)			
	Bond variance	**Foreign exchange variance**	**Stock variance**
Bond	-	0.0003690961	0.0024754751
Foreign exchange	0.0000000438	-	0.0000154207
Stock	0.0000000016	0.0000000936	-

The second highest volatility spillover channel is represented by the spillover from the bond market to the foreign exchange market. Similar to the case of the bond and the stock market, the asymmetry between the spillovers from the bond market to the foreign exchange market, compared to that from the foreign exchange market to the bond market is pronounced.[17] This result underlines the relative impact of volatility emanating within the market for fixed income securities on the dynamics in the other two major financial markets. Generally, the relatively strong volatility transition from the bond market returns to foreign exchange market returns suggests that an information event that alters the expectation about returns in the bond market will influence demand and trading in the foreign exchange market.

[17] The coefficient that reflects the extent of volatility spillovers from the foreign exchange market to the bond market is the second lowest volatility spillover coefficient (see Table 3).

The model results provide evidence of strong volatility spillovers from the foreign exchange market to the stock market. This spillover, which measures the extent to which an information event that alters the expectation about returns in the foreign exchange market will influence the demand and trading in the stock market, is the third largest among the cross-market volatility spillover effects. In contrast, the volatility spillover from the stock market to the foreign exchange market is the third lowest.

6.1 Impulse response analysis

A more complete understanding of volatility spillovers can be attained from an impulse response assessment of the various market volatilities. By applying the simple three variable vector autoregressive model, patterns in the volatility impulses can be observed[18]. Figure 1 depicts the response of bond market volatility to the one-day lag of the variances of the three major markets. The figure reflects the relatively significant one day lagged effect of bond market volatility on itself. This effect, however, dies out within two days of the initial shock. The figure suggests that the highest response of bond market volatility to foreign exchange market volatility occurs after two days. This effect dies out within six to seven days. The figure also reveals that stock market volatility, generally, exerts the least influence on bond market volatility over a ten-day observation range. This response peaks at around the third day and dies out after four to five days.

Figure 2 shows the response of foreign exchange market volatility to the lagged variances of the three major markets. The strong expectations component of foreign exchange market returns is reflected in the persistent five to six day impact of lagged foreign exchange return volatility on current volatility. The bond market volatility impact on foreign exchange market returns volatility dies out in two days. The stock market return volatility, however, has a lower but more sustained impact on returns volatility in the foreign exchange market. This impact dies out in approximately five to six days.

[18] In this application the impulses are standardized as a percentage of the initial common market effect of the market under consideration. Due to the mechanics of the model, it is highly unlikely that an initial common market impulse may evolve into a cross-market impulse in a future period. The reverse is also true.

Figure 1
Response of bond market variance to lagged market variances accounting for liquidity in the model

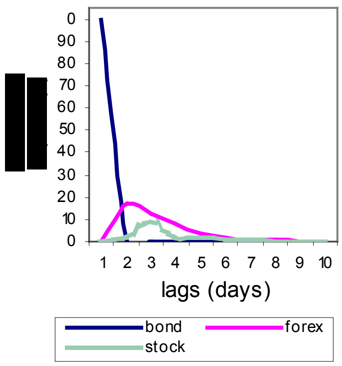


Figure 2
Rsponse of forex market variance to lagged market variances accounting for liquidity in the model

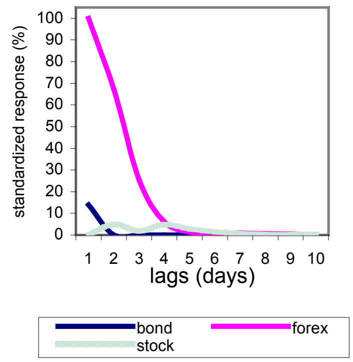


Figure 3
Response of stock market variance to lagged market variances accounting for liquidity in the model

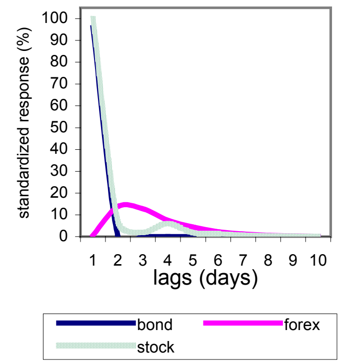


Figure 3 shows that the extent to which the lagged stock market variance influences the current variance falls off steeply over the first two days, but persist at a relatively low level before attaining a sub peak at about the fourth day after the initial shock. The impulse eventually dies out within five to six days. The diagram reflects the relatively brief response of stock market return volatility emanating from the bond market, dying out within two days of the initial shock. Although beginning at a relatively low value, the response of the stock market return volatility to that of the foreign exchange market intensifies to peak at approximately three days after the initial impulse. The impulse eventually dies out after six to seven days.

The above results suggest that, generally, volatility spillovers emanating from the bond market usually die out within two days. The brevity of this impulse indicates that in periods of bond market uncertainty, the existence of a fairly efficient private money market and the prudent base money management by the monetary authorities, provide strong support in curtailing the persistence of this uncertainty. Also, the relatively brief five to seven day and four to six day impulses that emanate from the foreign exchange and stock markets, respectively, indicate the relative effectiveness of monetary policy operations in curtailing uncertainty durations.

6.2 Liquidity impact

To measure the impact of the change in liquidity conditions, the model is re-run taking account of the maturity profile of Government and Central Bank fixed income securities, so as to capture the impact of new liquidity to the system. Table 4 reveals that accounting for liquidity effects results in a 0.02 per cent decline in the one-day bond market volatility momentum and an increase of 0.01 per cent in the one-day volatility momentum for both the foreign exchange and stock markets[19]. The results suggest that increased maturities have a positive impact on bond market volatility but a negative impact on the foreign exchange market and stock market volatilities.

Table 4
Factors that influence market volatilities accounting for liquidity in the model

Empirical volatility transmission channels			
Common market volatility			
	Bond variance	**Foreign exchange variance**	**Stock variance**
Bond	0.0022557325	-	-
Foreign Exchange	-	0.0026678871	-
Stock	-	-	0.0025815648
Cross market volatility (effect of squared residuals on variances)			
	Bond variance	**Foreign exchange variance**	**Stock variance**
Bond	-	0.0003708769	0.0024911296
Foreign Exchange	0.0000000439	-	0.0000155272
Stock	0.0000000016	0.0000000944	-

The difference in the results between Table 3 and Table 4 also indicates the liquidity impact on cross-market volatility spillovers. Notably, the introduction of liquidity had the smallest impact on the volatility spillovers to the bond market. New maturities resulted in a 0.23 per cent increase in the spillover from the foreign exchange market to the bond market and no increase in transmission from the stock market to the bond market. There is a uniformed volatility impact on the magnitude of volatility spillovers to the stock market. The introduction of liquidity in the financial system leads to a 0.63 per cent increase in the extent to which information events concerning the bond market spillover

[19] Generally, all the liquidity effects are low.

to impact the volatility in the stock market, compared to a 0.69 increase in the spillover from the foreign exchange market to the stock market. The liquidity impact on volatility spillovers to the foreign exchange market was varied. The introduction of liquidity results in a 0.85 per cent increase in the spillover from the stock market to the foreign exchange market and a 0.48 per cent increase in the extent of spillovers from the bond market to the foreign exchange market.

6.2.1 Liquidity impact and Impulse Response

Figure 4
Response of bond market variance to lagged market variances not accounting for liquidity in the model

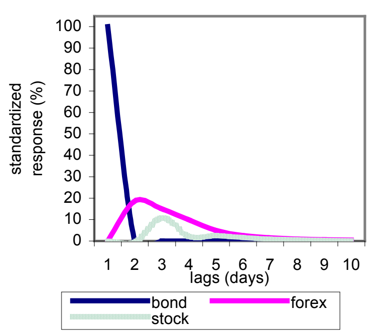


Figure 5
Response of forex market variance to lagged market variances not accounting for liquidity in the model

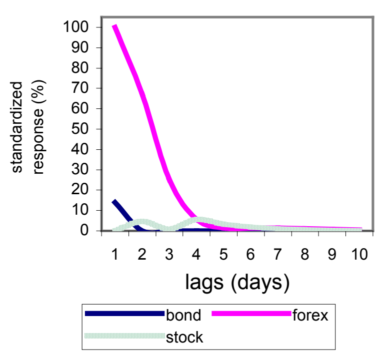
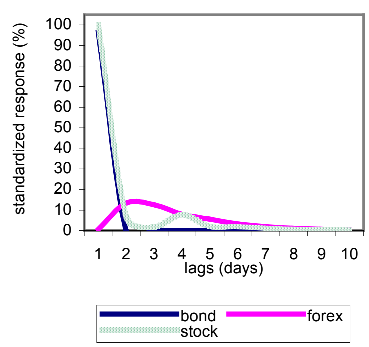


Figure 6
Response of stock market variance to lagged market variances not accounting for liquidity in the model



Figures 4, 5 and 6 show that there are considerable similarities between the impulses in the absence of the liquidity impact and with liquidity impact. These similarities imply that, although most of the initial volatility spillovers reflect slight differences with the consideration of liquidity in the financial system, the responses on the market volatilities considered over a ten-day range are hardly affected by this liquidity. This may be indicative of the extent to which monetary authorities employ effective policies to limit the occurrence of second round spillovers and hence greater impulses over higher lags.

7. Conclusion

In summary, the results of the model indicate that there are generally high levels of common market returns volatility relative to cross-market spillovers, within the Jamaican financial system. Of the three markets, the foreign exchange market exhibits the most pronounced common market volatility spillovers, followed by the stock market. The strong common market spillover in these two markets, relative to that of the bond market reflects the uncertainty momentum that often characterises these risky markets.

The cross-market volatility spillovers reflect the propagation of shocks from one financial market to another. The strongest cross-market effect occurs from the bond market to the stock market. This result is in sharp contrast to the weak volatility spillover from the stock market to the bond market. The second highest source of cross-market volatility propagation occurs from the bond market to the foreign exchange market. The relatively low spillover from foreign exchange market to the bond market also suggests some degree of asymmetry between the sequential volatility impacts between the two markets. The third highest volatility spillover channel is represented by volatility propagation from the foreign exchange market to the stock market. This channel reflects the lowest case of asymmetry. Considering the different magnitudes of spillover events, impulse response analyses reveal that the volatility spillovers from the bond market usually die out within two days, those from the foreign exchange market die out within five to seven days and spillovers emanating from the stock market usually take four to six days to die out. Similar to the case of common market volatility spillovers, this result also indicates that investors perceive the bond market to be more stable than the foreign exchange and stock markets.

The introduction of liquidity effects in the model suggests trivial differences in the various volatility spillover channels. Generally, the model indicates that the introduction of liquidity in the financial system causes a decline in the common market volatility spillover exhibited in the bond market. On the other hand, the liquidity causes an increase in foreign exchange market and stock market returns volatility. In terms of the cross-market spillover effects, changes in the liquidity conditions have a lesser impact on spillovers to the bond market than for the foreign exchange and the stock markets. Generally, the changes in liquidity do not have a significant impact on the duration of the volatility spillovers. This result suggests that monetary policy is successful in restricting the impact of volatility impulses within and between markets that stem from liquidity effects.

7.1 Policy recommendations

The results from this study highlight the complex nature of the Jamaican financial system. Each of the three major Jamaican financial markets is characterised by quantifiable uncertainty linkages with each other. Empirical results indicate that financial system participants, including the regulators, must consider the intricate market connections. These involve uncertainty spillovers that are relevant to portfolio profit maximization, monetary transmission and financial system stability.

With regard to the stability of the financial system, market risk is of particular importance. The market risk to which investors are exposed is highly dependent on the volatility, as well as the co-volatility in the different markets. Increased volatility in a single market return, by itself, may not pose a serious threat to the investor's portfolio as effective diversification strategies may help to minimize the increase in risk exposure. A more significant threat to systemic financial stability may be the existence of volatility linkages that may cause difficulties in diversifying portfolio risks[20]. These linkages increase the probability of systemic instability and therefore must be monitored by financial system regulators. The results from the study suggest that, based on the volatility linkages between the major financial markets, risk-based capital requirements for financial entities should be computed with consideration of the correlations between the different sources of market risk. The recent amendment to the BOJ Act that accounts for foreign exchange risk in the computation of capital requirements does not consider the linkage between the equity, foreign exchange and bond markets.[21] A more dynamic approach in accounting for market risk would involve simultaneous consideration for other types of market risks as suggested by the Basle II capital requirements.

With regard to monetary policy, the results reveal that the foreign exchange market exhibits the strongest and most sustained common market volatility spillover effects.

[20] Negative asset return correlation may be associated with positive volatility spillovers.
[21] This law was passed in October 2003.

Given the importance of foreign exchange market stability in attaining overall price stability in the Jamaican economy, the results support continuous emphasis on curtailing foreign exchange market uncertainties as quickly as possible. Interestingly, the model also reveals that even though the initial spillovers from the bond market are usually significant, these spillovers usually die out in a short time. This result indicates that the pursuit of foreign exchange market stability could take place at the expense of short-term disruptions in the bond market.

Importantly, the study highlights the need for the integration of monetary policy and financial system supervision. Specifically, monetary authorities must be aware of the volatility spillover implications of policy actions that may complicate the nature of market risks faced by institutional investors. The existence of volatility spillovers that obstruct the efficient operation of key institutional portfolio holders may result in the relative ineffectiveness of monetary policy during critical periods. Also, financial system regulators need to be apprised of pending changes to monetary policy, as this information may aid the implementation of timely measures to secure the stability of the financial system.

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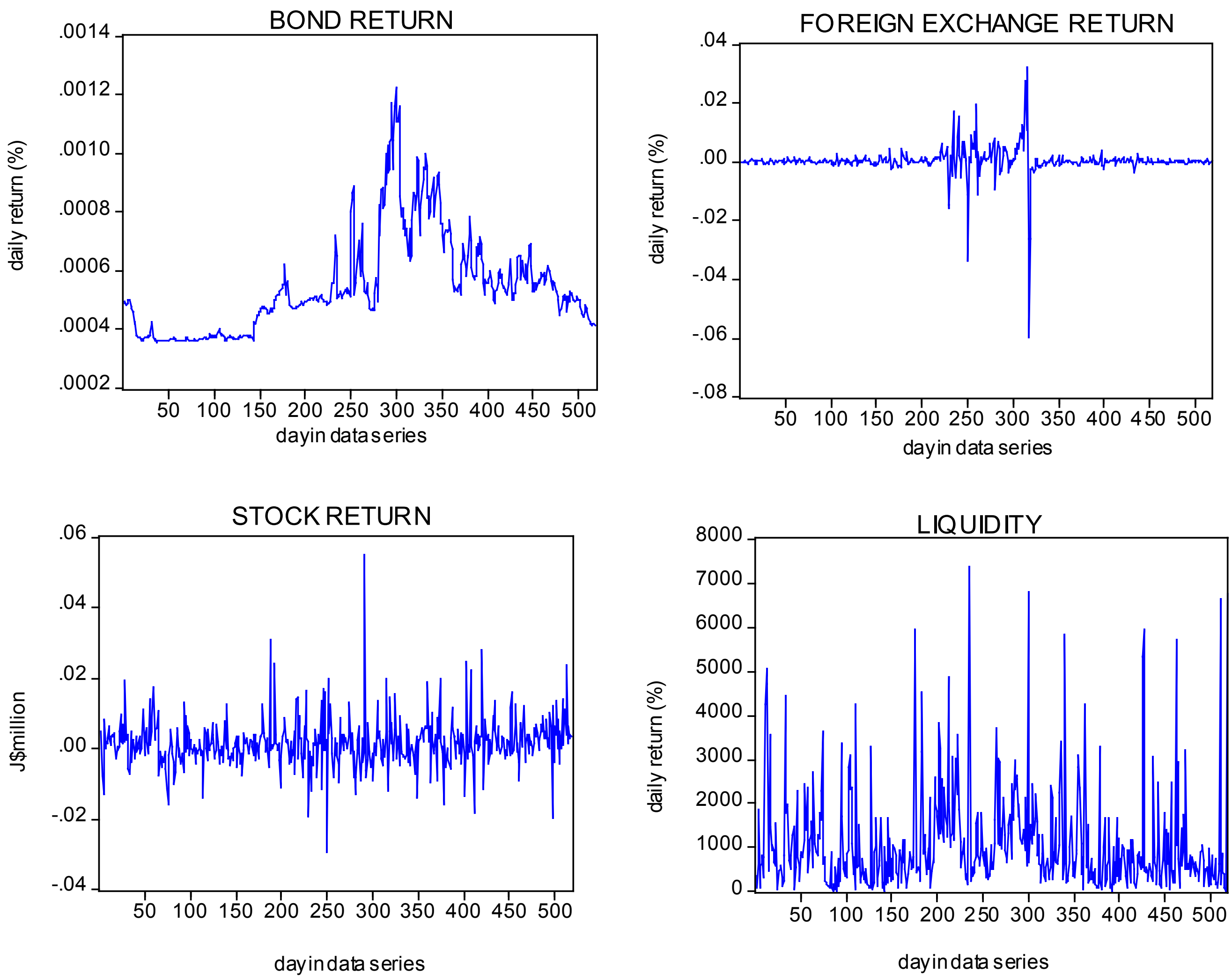
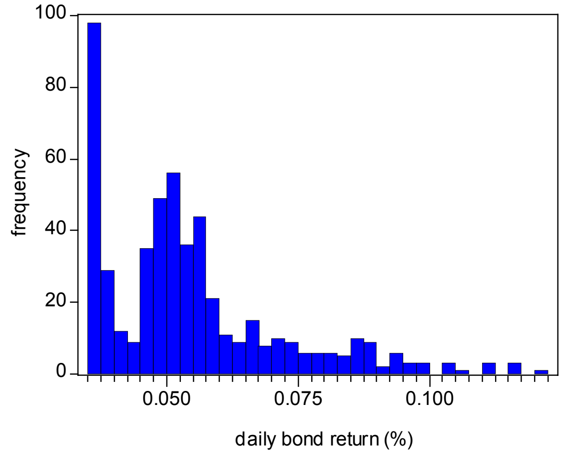
APPENDIX

Plot of daily return series



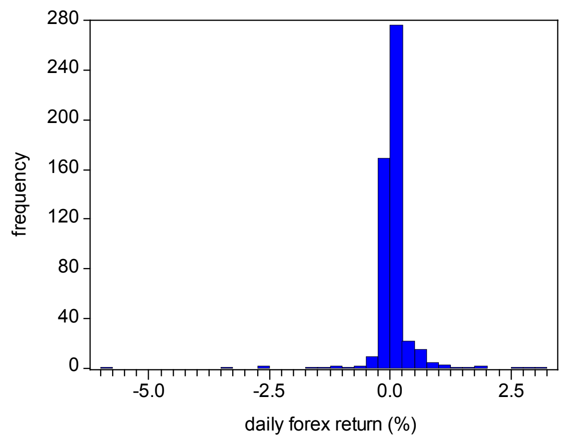
Histograms and Descriptive statistics

Figure A1 - Daily Bond Market Return
February 2002 to March 2004



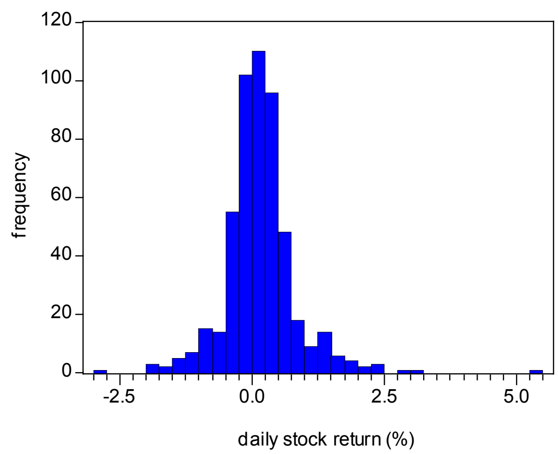
Series: BONDRET_PERCENT	
Sample 1 518	
Observations 518	
Mean	0.054698
Median	0.050991
Maximum	0.122458
Minimum	0.035841
Std. Dev.	0.017135
Skewness	1.308368
Kurtosis	4.583573
Jarque-Bera	201.9123
Probability	0.000000

Figure A2 - Daily Foreign Exchange Market Return
February 2002 to March 2004



Series: FOREXRET_PERCENT	
Sample 1 518	
Observations 517	
Mean	0.048332
Median	0.024647
Maximum	3.208311
Minimum	-5.952143
Std. Dev.	0.492645
Skewness	-3.413129
Kurtosis	59.24877
Jarque-Bera	69159.98
Probability	0.000000

Figure A3 - Daily Stock Market Return
February 2002 to March 2004



Series: STOCKRET_PERCENT	
Sample 1 518	
Observations 517	
Mean	0.164997
Median	0.142699
Maximum	5.498332
Minimum	-2.935986
Std. Dev.	0.689315
Skewness	1.179774
Kurtosis	11.99783
Jarque-Bera	1863.967
Probability	0.000000

Covariances derived from the BEKK representation of the Multivariate GARCH (1,1)

$$
\begin{aligned}
\sigma_{12,t} = c_{21}c_{22} + c_{31}c_{32} + a_{12}\left(a_{11}\varepsilon_{11,t-1} + a_{21}\varepsilon_{21,t-1} + a_{31}\varepsilon_{31,t-1}\right) &+ \\
a_{22}\left(a_{11}\varepsilon_{12,t-1} + a_{21}\varepsilon_{22,t-1} + a_{31}\varepsilon_{32,t-1}\right) &+ \\
a_{32}\left(a_{11}\varepsilon_{13,t-1} + a_{21}\varepsilon_{23,t-1} + a_{31}\varepsilon_{33,t-1}\right) &+ \\
b_{12}\left(b_{11}\sigma_{11,t-1} + b_{21}\sigma_{21,t-1} + b_{31}\sigma_{31,t-1}\right) &+ \\
b_{22}\left(b_{11}\sigma_{12,t-1} + b_{21}\sigma_{22,t-1} + b_{31}\sigma_{32,t-1}\right) &+ \\
b_{32}\left(b_{11}\sigma_{13,t-1} + b_{21}\sigma_{23,t-1} + b_{31}\sigma_{33,t-1}\right) &
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{13,t} = c_{31}c_{33} + a_{13}\left(a_{11}\varepsilon_{11,t-1} + a_{21}\varepsilon_{21,t-1} + a_{31}\varepsilon_{31,t-1}\right) &+ \\
a_{23}\left(a_{11}\varepsilon_{12,t-1} + a_{21}\varepsilon_{22,t-1} + a_{31}\varepsilon_{32,t-1}\right) &+ \\
a_{33}\left(a_{11}\varepsilon_{13,t-1} + a_{21}\varepsilon_{23,t-1} + a_{31}\varepsilon_{33,t-1}\right) &+ \\
b_{13}\left(b_{11}\sigma_{11,t-1} + b_{21}\sigma_{21,t-1} + b_{31}\sigma_{31,t-1}\right) &+ \\
b_{23}\left(b_{11}\sigma_{12,t-1} + b_{21}\sigma_{22,t-1} + b_{31}\sigma_{32,t-1}\right) &+ \\
b_{33}\left(b_{11}\sigma_{13,t-1} + b_{21}\sigma_{23,t-1} + b_{31}\sigma_{33,t-1}\right) &
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{21,t} = c_{22}c_{21} + c_{32}c_{31} + a_{11}\left(a_{12}\varepsilon_{11,t-1} + a_{22}\varepsilon_{21,t-1} + a_{32}\varepsilon_{31,t-1}\right) &+ \\
a_{21}\left(a_{12}\varepsilon_{12,t-1} + a_{22}\varepsilon_{22,t-1} + a_{32}\varepsilon_{32,t-1}\right) &+ \\
a_{31}\left(a_{12}\varepsilon_{13,t-1} + a_{22}\varepsilon_{23,t-1} + a_{32}\varepsilon_{33,t-1}\right) &+ \\
b_{11}\left(b_{12}\sigma_{11,t-1} + b_{22}\sigma_{21,t-1} + b_{32}\sigma_{31,t-1}\right) &+ \\
b_{21}\left(b_{12}\sigma_{12,t-1} + b_{22}\sigma_{22,t-1} + b_{32}\sigma_{32,t-1}\right) &+ \\
b_{31}\left(b_{12}\sigma_{13,t-1} + b_{22}\sigma_{23,t-1} + b_{32}\sigma_{33,t-1}\right) &
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{23,t} = c_{32}c_{33} + a_{13}\left(a_{12}\varepsilon_{11,t-1} + a_{22}\varepsilon_{21,t-1} + a_{32}\varepsilon_{31,t-1}\right) &+ \\
a_{23}\left(a_{12}\varepsilon_{12,t-1} + a_{22}\varepsilon_{22,t-1} + a_{32}\varepsilon_{32,t-1}\right) &+ \\
a_{33}\left(a_{12}\varepsilon_{13,t-1} + a_{22}\varepsilon_{23,t-1} + a_{32}\varepsilon_{33,t-1}\right) &+ \\
b_{13}\left(b_{12}\sigma_{11,t-1} + b_{22}\sigma_{21,t-1} + b_{32}\sigma_{31,t-1}\right) &+ \\
b_{23}\left(b_{12}\sigma_{12,t-1} + b_{22}\sigma_{22,t-1} + b_{32}\sigma_{32,t-1}\right) &+ \\
b_{33}\left(b_{12}\sigma_{13,t-1} + b_{22}\sigma_{23,t-1} + b_{32}\sigma_{33,t-1}\right) &
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{31,t} = c_{33}c_{31} + a_{11}\left(a_{13}\varepsilon_{11,t-1} + a_{23}\varepsilon_{21,t-1} + a_{33}\varepsilon_{31,t-1}\right) &+ \\
a_{21}\left(a_{13}\varepsilon_{12,t-1} + a_{23}\varepsilon_{22,t-1} + a_{33}\varepsilon_{32,t-1}\right) &+ \\
a_{31}\left(a_{13}\varepsilon_{13,t-1} + a_{23}\varepsilon_{23,t-1} + a_{33}\varepsilon_{33,t-1}\right) &+ \\
b_{11}\left(b_{13}\sigma_{11,t-1} + b_{23}\sigma_{21,t-1} + b_{33}\sigma_{31,t-1}\right) &+ \\
b_{21}\left(b_{13}\sigma_{12,t-1} + b_{23}\sigma_{22,t-1} + b_{33}\sigma_{32,t-1}\right) &+ \\
b_{31}\left(b_{13}\sigma_{13,t-1} + b_{23}\sigma_{23,t-1} + b_{33}\sigma_{33,t-1}\right) &
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{33,t} = c_{33}c_{32} + a_{12}\left(a_{13}\varepsilon_{11,t-1} + a_{23}\varepsilon_{21,t-1} + a_{33}\varepsilon_{31,t-1}\right) &+ \\
a_{22}\left(a_{13}\varepsilon_{12,t-1} + a_{23}\varepsilon_{22,t-1} + a_{33}\varepsilon_{32,t-1}\right) &+ \\
a_{32}\left(a_{13}\varepsilon_{13,t-1} + a_{23}\varepsilon_{23,t-1} + a_{33}\varepsilon_{33,t-1}\right) &+ \\
b_{12}\left(b_{13}\sigma_{11,t-1} + b_{23}\sigma_{21,t-1} + b_{33}\sigma_{31,t-1}\right) &+ \\
b_{22}\left(b_{13}\sigma_{12,t-1} + b_{23}\sigma_{22,t-1} + b_{33}\sigma_{32,t-1}\right) &+ \\
b_{32}\left(b_{13}\sigma_{13,t-1} + b_{23}\sigma_{23,t-1} + b_{33}\sigma_{33,t-1}\right) &
\end{aligned}
$$