Principal Component Value at Risk: an application to the measurement of the interest rate risk exposure of Jamaican Banks to Government of Jamaica (GOJ) Bonds

Mark Tracey[1]

Financial Stability Department
Research & Economic Programming Division
Bank of Jamaica

Abstract

This paper develops an effective value at risk (VaR) methodology to complement existing Bank of Jamaica financial stability assessment tools. This methodology employs principal component analysis and key rate durations for assessing interest rate risk of the Jamaican banking sectors' holdings of both local and global Government of Jamaica (GOJ) bonds. Principal Components Analysis (PCA) is proposed as a tractable and simple-to-implement method for extracting market risk factors from observed data. This approach, which is informationally efficient, quantifies the risk associated with portfolios using three principal factors that affect yield curves. Due to the orthogonal nature of the factors, correlation and covariance between the yields do not have to be explored, simplifying the calculation of VaR for the portfolios. Results of this paper indicate that the PC VaR outturn for the Jamaican banking system is higher relative to both parametric and historical VaR outturns suggesting that the PC VaR holds more information as it relates to the risks impacting banking system portfolios.

JEL Classification: G11, G28, G32

Keywords: Key Rate Duration, Principal Component Analysis, Value at Risk

[1] The views expressed are those of the author and do not necessarily reflect those of the Bank of Jamaica.

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1. Introduction

The main contribution of this paper is the development of an additional Value at Risk (VaR) [2] framework for measuring and monitoring risk, contingent on changes in the interest rate term structure of Government of Jamaica (GOJ) bonds. The paper employs principal component analysis (PCA) for the efficient evaluation of the risk factors which affect bond returns in a VaR framework. The paper takes into consideration both the skewness and fat tailed nature associated with the return on bonds. It is widely understood that returns on financial instruments tend to systematically depart from normality, with financial returns showing higher peaks and fatter tails than normal distributions, especially over shorter periods. Thus a framework, which imposes normality as being implicit in the evolution of returns of financial instruments, such as the parametric-VaR, would consistently underestimate the risk of loss to a portfolio. That is, extreme events happen more frequently than observed under normal conditions, hence underscoring the importance of a framework which would address this particular stylized fact that financial data presents.

Principal Components Analysis, which is a widely used technique in portfolio risk management, reduces the amount of risk factors driving a portfolio re-evaluation and can be combined with key rate duration to calculate principal component duration (PCD) factors. "Risk factors" are often defined and used to summarize observed changes in market prices and volatilities. Key rate duration (KRD), which is a risk metric applied to bond prices, measures the sensitivity of a security's value to a 1.0 per cent change in yield for a given maturity.[3] It allows for the examination of changes in the yield curve as a result of non-parallel movement in interest rates. The overall advantage of KRD is that it gives information on whether or not the portfolio is exposed to risk from non-parallel shifts in the yield curve, such as steepening or flattening, which cannot be measured from

[2] Value at risk may be defined as the worst loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger (Jorion 3rd ed.)
[3] See Nawalkha 2005.

dollar duration.[4] The advantage of combining these two concepts is that it allows risk managers to benefit from the intuitive description of risk provided by KRD as well as the computational benefits associated with PCA. It also fosters more parsimonious portfolio strategies that do not exhaust all the degrees of freedom in portfolio construction.[5]

Principal component analysis was first applied to the fixed income market in a paper by Garbade (1997). The principal component (PC) is a linear combination of the original variables of interest and the components are orthogonal resulting in them being additive and statistically independent.[6] Thus, the sensitivity of fixed term instruments to movements in any of the PCs can be evaluated. In short, PCA finds a linear combination of the observed asset returns that "explains" as much as possible the observed variability of the data. The first variable explains the greatest amount of variation, the second component defines the next largest and is independent to the first PC, and so on and so forth.

When PCA is applied to the term structure of interest rate, a fairly standard result would provide three PC values that would be able to explain the majority of the total variation of entire yield. The first PC generally explains up to 80.0 per cent of change in the yield curve while the second and third PCs would explain 11.0 per cent and 5.0 per cent, respectively.[7] The first PC value would be able to explain the impact of parallel shifts of the yield curve; the second PC generates an interpretation of the “tilt” or “rotation” of the curve while the third indicates the “twist”. Parallel movements of the yield curve are commonly caused by expected inflation, a change in slope may be caused by changes in expected long-term inflation or changes in market risk premiums, while changes in the curvature of the curve represents changes in the volatility of interest rates.[8]

[4] Government of Canada Treasury Risk Management Framework.
[5] See Nawalkha, 2005.
[6] For risk management purposes, additively is important because it allows evaluation of the impact of say one unit of added parallel shift risk to an existing position. Statistical independence is important because it allows the factors to be managed separately, say to hedge a parallel shift without having to think about its effect on the other factors (Nifhker, 2000).
[7] See Falkenstein, 1997.
[8] It must be noted that short-term instruments tend to be more sensitive to parallel movements of the yield curve while instruments with long term tenure will be sensitive to changes in the slope of the curve.

Jatnshidian and Zhu (1997) applied PCA to fixed-income portfolios in order to derive a discrete approximation of the portfolio value distribution, while Loretan (1997) and Frye (1997) apply it in the context of a VAR methodology. The advantage of these frameworks emanates from its characteristics of analyzing risk associated with investment products without worrying about the normality issues associated with the data as well as information lost due to correlation and covariance issues. It also allows researchers the power of assessing components of risk in isolation, therefore leading to a proper assessment of individual risk components on portfolio valuation which can be readily investigated.

The main focus of the paper will be on assessing the risk associated with the GOJ bond component of the banking system bond portfolio using a PC VAR framework. Focus will only be placed on GOJ bonds due to a large portion of the banks' bond portfolio consisting of GOJ bonds as at end December 2008. This holds true particularly for building societies (see table 1).

Table 1

HOLDINGS OF GOVERNMENT OF JAMAICA BONDS BY BANKING SECTOR AS AT DECEMBER 2008				
	Dom.Securities as % of Total Assets	Glob.Securities as % of Total Assets	Dom.Securities as % of Total Bond Portfolio	Glob.Securities as % of Total Bond Portfolio
CBank1	15.0%	0.00%	29.1%	0.00%
CBank2	6.5%	0.00%	15.3%	0.00%
CBank3	3.1%	0.00%	28.3%	0.00%
CBank4	8.0%	0.09%	33.4%	0.37%
CBank5	15.0%	0.16%	40.4%	0.42%
CBank6	11.3%	0.02%	79.6%	0.17%
CBank7	7.8%	0.01%	64.5%	0.07%
Mbank1	3.1%	0.58%	5.9%	1.09%
Mbank2	1.9%	0.11%	17.1%	1.03%
Mbank3	0.9%	0.14%	6.2%	1.01%
BSoc1	0.0%	0.00%	0.0%	0.00%
BSoc2	9.8%	0.04%	64.2%	0.26%
BSoc3	6.2%	0.09%	36.1%	0.53%
BSoc4	14.5%	0.00%	84.7%	0.00%

The remainder of this paper is divided into six sections. Section two will give information on the data to be used, while section three will present the procedures used in this study. Section four and five will present the results and discussions of the results, respectively. Section six will give some conclusions and recommendations arrived at from the study.

2. Data Set

The data consists of a time series of yields from 23 February 2006 to 18 March 2009 (796 data points) for GOJ global securities with maturity structures of 7-year, 9-year, 20-year and 30-year. In addition, GOJ domestic bond yields ranging from 3 January 2008 to 18 March 2009 (273 data points) with maturity structures of 6-month, 2-year, 3-year, 6-year, 9-year, 15-year, 20-year and 25-year.[9] Natural logarithms of these series were taken and then the series were first-differenced to induce stationarity. Holdings of GOJ securities by each institution were obtained from re-pricing data for the banking system as at end-December 2008.

3. Methodology

3.1 Statistical Analysis of data

The properties of the log of interest rate changes were observed to ascertain if term structures followed the stylized facts known in other markets.[10] Simple statistical tests were conducted on the data as well as the Augmented-Dickey Fuller test to ascertain if the data was stationary, as well as Jarque-Bera test for normality. [11]

3.2 Determining the Key Rate Duration

Each bond's repricing structure and scheduled coupon payments were used to compute the yield to maturity per instrument. Equation (1) was then employed to ascertain the KRD of each bond.

$$KRD(i) = \frac{1}{p}\frac{CF_i \times t_i}{e^{t_i \times y(t_i)}} \qquad (1)$$

Where $KRD(i)$ is the ith key rate duration, p is the price of the bond, CF_i is the ith cash flow, and t_i is the ith time period.

3.3 Determining the Principal Components and Principal Components Duration

Consider a set of N variables $y_1,...,y_n$ of changes in bond yields with covariance matrix Σ. We wish to reduce the dimensions of Σ without too much loss of content, by approximating it by another matrix Σ^*. The goal is to provide a good approximation of

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[9] Yield data was obtained from Bloomberg.
[10] If the yield curve follow a specific pattern this can be used to find specific functional forms matching the curve.
[11] See appendix 1.

the variance of a portfolio $Z = w' y$ using $V^* Z = w' \Sigma^* w$. The process consists of replacing the original variables y by another set, Δc , suitably selected.

The first PC is the linear combination

$$\Delta c = \sum_{i=1}^{m} u_{ji} \Delta y(t_i) \qquad j = 1,......, m \qquad (2)$$

such that its variance is maximized, subject to a normalization constraint on the norm of the factor exposure vector $m_1' m_1 = 1$. A constrained optimization of this variance, $s^2(\Delta c_1) = m_1' \Sigma m_1$, shows that the vector m_1 must satisfy $\Sigma m_1 = l_1 m_1$. Here, $s^2(\Delta c_1) = l_1$ is the largest eigenvalue of the matrix Σ and m_1 its associated eigenvector.

The second PC is the one that has greatest variance subject to the same normalization constraint $m_2' m_2 = 1$ and to the fact that it must be orthogonal to the first $m_2' m_1 = 0$. And so on for all the others.

This process basically replaces the original set of variables, y, by another set of orthogonal factors that has the same dimension but where the variables are sorted in order to decreasing importance. This leads to the singular value decomposition, which decomposes the original matrix as

$$\Sigma = PDP' = [m_1 ... m_n] \begin{bmatrix} l_1 & ... & 0 \\ . & & . \\ 0 & ... & l_N \end{bmatrix} \begin{bmatrix} m_1' \\ . \\ . \\ . \\ m_n' \end{bmatrix} \qquad (3)$$

where P is an orthogonal matrix, i.e., such that its inverse is also its transpose, $PP' = I$ and D a diagonal matrix composed of the $l_1's$. [12]

[12] Matlab code for generating PC is given in appendix 4.

The benefit of this approach is that we can now simulate movements in the original variables by simulating movements with a much smaller set of PCs. It must be noted that the change in interest rates can now be expressed by equation (5) below

$$\Delta y(t_i) = l_{ih}\Delta c_h + l_{is}\Delta c_s + l_{ic}\Delta c_c \tag{4}$$

Equation (5) describes the change in interest rates as a function of the product of the factor loadings and the principal components.

The principal components are defined as follows:

$$\Delta c_h = \Delta c_1^* = \frac{\Delta c_1}{\sqrt{l_1}}, \qquad \Delta c_s = \Delta c_2^* = \frac{\Delta c_2}{\sqrt{l_2}}, \qquad \Delta c_c = \Delta c_3^* = \frac{\Delta c_3}{\sqrt{l_3}}, \tag{5}$$

The factor loadings are defined as follows:

$$l_{ih} = u_1\sqrt{l_1}, \qquad l_{is} = u_{2i}\sqrt{l_2}, \qquad l_{ic} = u_{3i}\sqrt{l_3}, \tag{6}$$

$l_1 \geq l_2 ... \geq l_n$ are the eigenvalues of c, ranked in decreasing order and, $u_1, m_2, ..., m_n$ are the corresponding eigenvectors.[13]

The first three PCs chosen to model the yield curve dynamics should (theoretically) explain almost 90% of the variation in yields. The following criterion is usually used to ascertain the number of PCs to be adopted,

$$\frac{l_1 + ... + l_k}{l_1 + ... + l_n} > 1 - e^* \tag{7}$$

[13] See appendix 2

The PCs with indices j>k have a small effect on the underlying vector of risk factors since the corresponding have eigenvalues which are small. This criterion specifies the ratio of the total variances of $\hat{x}$ and x is given as,

$$VaR(x) = \sum_{i} Var(x) = E\|x\|^2 = E\|x^T.x\| \tag{8}$$

$$= \sum_{j=1}^{k} l E\left(h_i^2\right)(U_i^T, U_i) = \sum_{i=1}^{k} l_i$$

Once the PCs have been identified, the PCD are computed using the equation (10) below:

$$PCD(v) = \sum_{i=1}^{m} KRD(i) \times l_{iv} \tag{9}$$

Equation (10) indicates that the PCD is summation of the product of the KRDs matrix and factor loadings matrix of each bond. Note that v indicates whether the height, slope or curvature is being calculated for each bond, which correspond to parallel shifts, tilts and changes in the curvature of the yield curve, respectively.

The portfolio can be immunized using the PC model.[14] The immunization constraint is given as follows:

$$PCD(h) = p_1 \times PCD_1(h) + p_2 \times PCD_2(h) + ...p_n \times PCD_n(h) = H \times l_{Hh} \tag{10}$$

$$PCD(s) = p_1 \times PCD_1(s) + p_2 \times PCD_2(s) + ...p_n \times PCD_n(s) = H \times l_{Hs}$$

$$PCD(c) = p_1 \times PCD_1(c) + p_2 \times PCD_2(c) + ...p_n \times PCD_n(c) = H \times l_{Hc}$$

[14] Immunization may be defined as portfolio diversification (Nawalkha, 2005).

$$p_1 + p_2 + \ldots . p_n = 1$$

Where p represents the proportion of various types of bonds held in the portfolio.

This immunization method is used to construct domestic and global GOJ instrument portfolios for the local banking sector using the actual domestic as well as actual US$ denominated re-pricing schedule as at end-December 2008.

3.4 Determining VaR Using Principal Component Duration

The VaR value at the 99th per cent levels for each portfolio using the PCD model was then calculated using the following equation,

$$VaR_{99} = \Gamma \times 2.326 \times \sqrt{PCD_{port}(h)^2 + PCD_{port}(s)^2 + PCD_{port}(c)^2} \qquad (11)$$

Where Γ is the market value of the portfolio and the 99th percentile of a standard distribution is 2.326.

A 10-day PC VaR was calculated as well as a corresponding 10-day parametric and non-parametric VaR for comparative purposes.

4. Results

4.1 Statistical Analysis

The statistical properties observed for interest rate term structure of domestic GOJ bonds as well as global GOJ bonds were consistent with stylized facts known to emerging market bonds. Bond yields moved around a long-term average and exhibited large volatility during periods of uncertainty. This was particularly true for global bonds (see Figure 1 and Figure 2).

Figure 1.

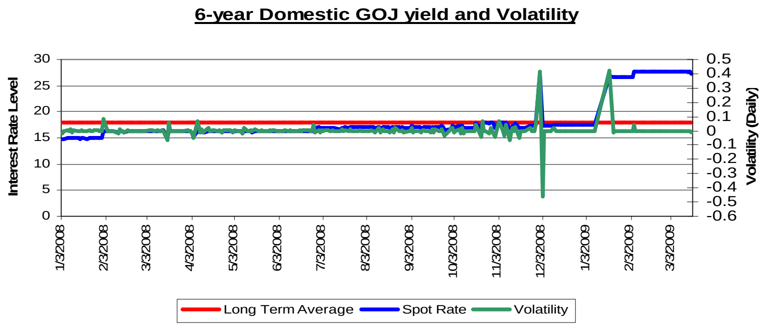
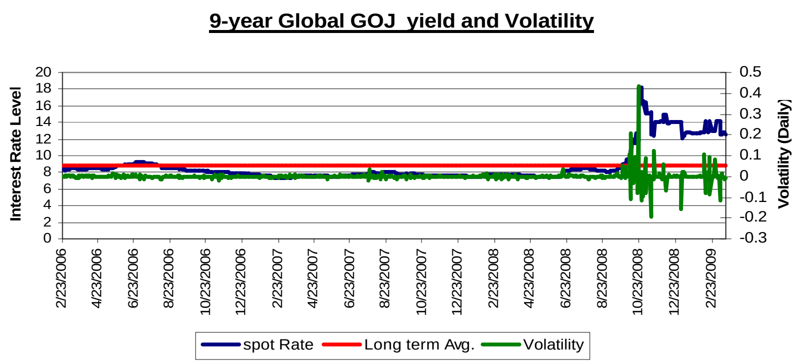


Figure 2.



The application of the Dickey-Fuller unit root test indicated that the log of interest rate changes does not have a unit root and is therefore stationary.[15] The stationarity of the data allows for constant autocorrelation over time which permits for the efficient employment of PCA. The results of the normality test conducted indicate that the returns for all bonds are not normally distributed.[16] Taken together, these results suggests that the use of PC-VaR rather than the parametric VaR would be best suited for the evaluation of the susceptibility of portfolio values to changes in the term structure of the yield curve of GOJ bonds.

[15] See appendix 1 for results of Augmented Dickey-Fuller test.
[16] See appendix 1 for results of Jarque-Bera test.

4.2 Examination of Re-pricing Gap

An examination of re-pricing schedule for domestic assets structure in the banking sector as at end-December 2008 reveals that commercial banks as well as building societies held a larger portion of their assets in domestic instruments (see Table 2). The investment profile of, commercial bank 7 and merchant bank 1 followed a barbell distribution.[17] The investment profiles for commercial bank 1, commercial bank 2, commercial bank 3, commercial bank 4, commercial bank 5, commercial bank 6, merchant bank 2, building society 2, building society 3 and building society 4 followed a bullet distribution.[18]

For global bond holdings, commercial bank 5, commercial bank 7, merchant bank 3 and followed a barbell investment profile. Commercial bank 4, commercial bank 6, merchant bank 2, building society 2 and building society 3 followed a bullet investment profile (see Table 2).

[17] A barbell portfolio has high key rate durations corresponding to the short and long term interest rates and low durations for intermediate rates, and so it is preferred if the short and the long rates fall more than the intermediate rates (Nawalkha, 2005).

[18] A bullet portfolio has low key rate durations corresponding to the short and long term interest rates and high durations for intermediate rates, and so it is preferred if the short and the long rates fall less than the intermediate rates (Nawalkha, 2005).

Table2.

The Re-pricing gap Domestic Assets Structure (end-December 2008).							
	91 - 365 days	**1 - 2 yrs**	**2 - 5 yrs**	**5 - 10 yrs**	**10 - 15 yrs**	**15 - 20 yrs**	**over 20 yrs**
			Commercial Banks				
CBank 1	12.90%	37.56%	36.60%	6.84%	4.38%	0.00%	1.72%
CBank 2	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CBank 3	0.00%	7.56%	82.48%	9.96%	0.00%	0.00%	0.00%
CBank 4	64.98%	0.00%	35.02%	0.00%	0.00%	0.00%	0.00%
CBank 5	84.89%	0.12%	10.58%	2.23%	0.32%	1.86%	0.00%
CBank 6	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CBank 7	38.82%	0.00%	51.20%	8.15%	1.82%	0.00%	0.00%
			Merchant Banks				
Mbank 1	16.20%	34.70%	0.55%	7.63%	40.91%	0.00%	0.00%
MBank2	0.00%	32.65%	67.35%	0.00%	0.00%	0.00%	0.00%
MBank3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
			Building Societies				
BSoc1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
BSoc2	86.76%	11.70%	0.00%	1.53%	0.00%	0.00%	0.00%
BSoc3	98.02%	0.00%	1.98%	0.00%	0.00%	0.00%	0.00%
BSoc4	95.22%	0.00%	4.78%	0.00%	0.00%	0.00%	0.00%

The Re-pricing gap FX Assets Structure (end-December 2008).							
	91 - 365 days	**1 - 2 yrs**	**2 - 5 yrs**	**5 - 10 yrs**	**10 - 15 yrs**	**15 - 20 yrs**	**over 20 yrs**
			Commercial Banks				
CBank 1	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 2	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 3	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 4	0.000%	0.000%	73.886%	26.114%	0.000%	0.000%	0.000%
CBank 5	4.329%	4.136%	42.691%	29.459%	19.385%	0.000%	0.000%
CBank 6	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 7	5.887%	37.791%	0.000%	16.002%	40.320%	0.000%	0.000%
			Merchant Banks				
Mbank 1	0.000%	8.939%	1.039%	12.643%	12.623%	20.090%	44.665%
MBank2	0.000%	79.730%	20.270%	0.000%	0.000%	0.000%	0.000%
MBank3	24.176%	0.000%	57.497%	8.910%	9.416%	0.000%	0.000%
			Building Societies				
BSoc1	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
BSoc2	11.480%	74.568%	0.000%	13.952%	0.000%	0.000%	0.000%
BSoc3	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
BSoc4	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%

4.3 Sensitivity Factors of Instruments

The KRD outturn indicates that as the maturity of each bond in the portfolio structure increased, the KRD increased, consistent with *a priori* expectations. In addition, the duration and convexity results indicate that global bonds prices are more sensitive to interest rate movements relative to domestic bonds. It must be noted that most global bonds in the banks' portfolios were available for sale (see Table 3).

Table 3.

SENSITIVITY FACTORS OF INSTRUMENTS					
	Domestic		**Global**		
Maturity	**Duration**	**Convexity**	**Maturity**	**Duration**	**Convexity**
6 mm	0.5	0.250	**7 yr**	4.889	28.994
2 yr	1.8	3.401	**9 yr**	5.466	38.782
3 yr	2.433	6.561	**20 yr**	8.116	101.420
6 yr	3.908	19.006	**30 yr**	12.472	214.455
9 yr	4.884	32.352			
15 yr	5.714	50.612			
20 yr	6.164	62.733			
25 yr	7.093	82.544			

4.4 Principal Component Analysis

Domestic eigenvectors and eigenvalues of the covariance matrix indicate that the first three PCs explained 86.2 per cent of the variation in the term structure of the interest rate for domestic bonds. For global bonds, the first three principal components explained 95.3 per cent of variations in the term structure of the interest rates. Therefore, the first three components were sufficient in explaining the variations in interest rates over time across all bond term structures.[19]

[19] The number of relevant PCs (risk factors) is determined by the correlation structure of the data: if the data are all highly correlated, a few PCs are sufficient to explain most of the variation in the data.

Table 4.

PORTION OF VARIANCE EXPLAINED BY FIRST THREE PC						
	Domestic			**Global**		
	PC1	**PC2**	**PC3**	**PC1**	**PC2**	**PC3**
Eigenvalue	0.008	0.002	0.002	0.560	0.112	0.055
Variability (%)	59.761	14.484	12.002	73.482	14.646	7.212
Cumulative (%)	59.761	74.244	86.246	73.482	88.128	95.339

For domestic instruments, 59.8 per cent of the movement in term structures can be attributed to parallel shifts. The tilt in the curve accounts for 14.5 per cent of the curve movement. The third component which induces the curvature of the yield curve accounts for 12.0 per cent of the curve movement.[20]

The first component, with the exception of the 6-month bond, was positively correlated with rate changes. This indicates that the first PC represents parallel movement of the yield curve. The second PC, which is indicative of a tilt of the yield curve, was made evident as medium-term bonds showed a negative correlation with PC 2. The third PC exhibited correlation traits that signify a curvature in the yield curve (see Table 5 and Figure 3).

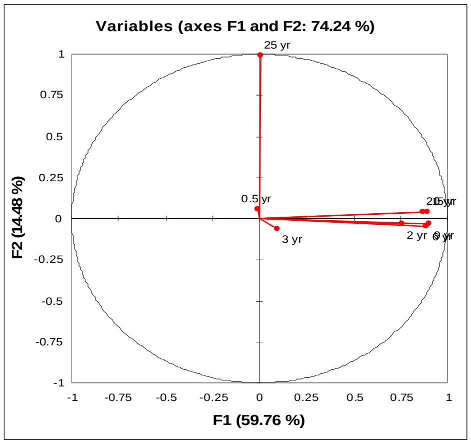
[20] See appendix 3.

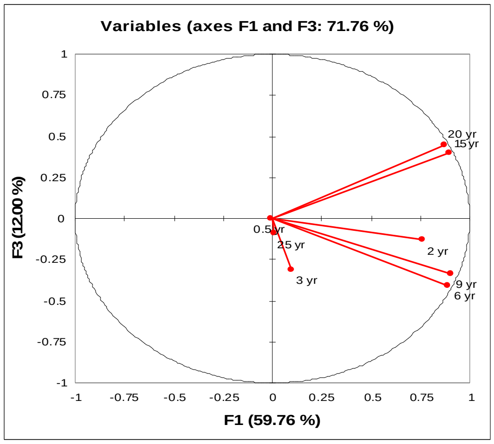
Table 5.

PCA ON DAILY BASIS: FACTOR LOADINGS							
	Domestic			Global			
Maturity	**PC1**	**PC2**	**PC3**	**Maturity**	**PC1**	**PC2**	**PC3**
6 m	-0.00005	0.00047	-0.00003				
2 yr	0.025	-0.001	-0.004				
3 yr	0.003	-0.002	-0.010				
6 yr	0.043	-0.002	-0.020	**7 yr**	-0.034	0.007	0.056
9 yr	0.042	-0.001	-0.016	**9 yr**	0.000	0.333	-0.022
15 yr	0.041	0.002	0.018				
20 yr	0.042	0.002	0.021	**20 yr**	0.002	0.031	0.226
25 yr	0.000	0.043	-0.004	**30 yr**	0.748	0.000	0.002

Figure 3

Diagrammatic representation of correlations between variables and factors for Domestic Bonds

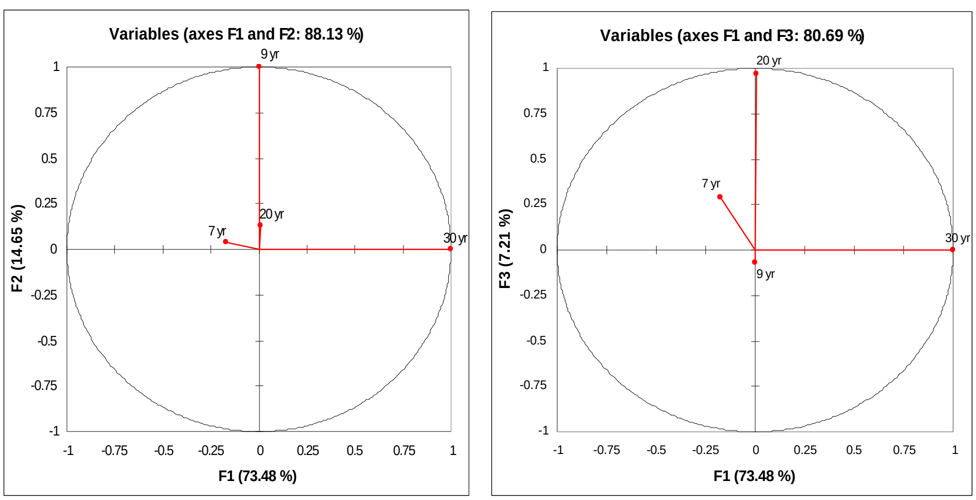




The results for the global bonds indicated that PC1 accounted for 73.5 per cent of movements in term structure. This was followed by PC2 which accounted for 14.6 per cent of movements. PC3 had a 7.2 per cent influence on the term structure. The first component with the exception of the 7-year bond was positively correlated with rate changes. The second PC had a positive relationship with term structure changes for all global bonds. The third PC showed a positive relationship with term structure changes with the exception of the 9-year global bond. The correlation outturn for each PC did not give conclusive evidence in the nature of movement by the yield curve for each PC which may be due to insufficient amount of global bonds (see figure 4).

Figure 4

Diagrammatic representation of correlations between variables and factors for Global Bonds



4.5 Value at Risk Outturn

The PC VaR outturn for the domestic portfolio indicated that the merchant banking sector had the highest risk for the banking sector. The 99^{th} percentile parametric and non-parametric VaR outturn for the domestic portfolio also indicated that the merchant banking sector had the highest risk for the banking sector. This was due mainly to merchant banks having a large proportion of their bonds as medium-term instruments.

Risk outturn for commercial banks was also high and was due to commercial banks holding a greater proportion of their domestic government bond investments in medium-term instruments. This was particularly true for CBank 1, CBank 3 and CBank 7.

Table 6.

Comparison of Different Risk Measures for Domestic GOJ Bonds: 10-day PC VaR versus 10-day VaR

	10-day Principal Component VaR	10-day VaR: Parametric VaR	10-day VaR: Non Parametric VaR
Commercial Banks			
CBank 1	-26.2%	-10.2%	-15.5%
CBank 2	0.0%	0.0%	0.0%
CBank 3	-31.4%	-14.4%	-25.7%
CBank 4	-2.6%	-2.0%	-2.4%
CBank 5	-4.8%	-2.6%	-5.3%
CBank 6	-0.1%	-0.3%	-0.4%
CBank 7	-7.7%	-3.6%	-7.0%
Merchant Banks			
Mbank 1	-39.6%	-26.9%	-17.4%
Mbank 2	-20.3%	-10.3%	-13.2%
Mbank 3	0.0%	0.0%	0.0%
Building Societies			
Bsoc 1	0.0%	0.0%	0.0%
Bsoc 2	-1.4%	-0.5%	-0.6%
Bsoc 3	-0.2%	-0.4%	-0.5%
Bsoc 4	-0.3%	-0.3%	-0.5%
Mean	-9.6%	-5.1%	-6.3%

The global bond PC VaR and 99th percentile parametric and non-parametric VaR outturn for merchant banks' foreign portfolio had the highest risk of exposure. The high risk associated with merchant banks' global GOJ bonds was due mainly to their holding of long-term global bonds with maturities of over 20 years which accounted for 42.0 per cent of their portfolio value. Merchant bank 1 had a large share of its investment in bonds over 20 years. Note however, merchant bank 1 had a higher 99th percentile parametric and non-parametric VaR outturn relative to its PC VaR outturn. This is due mainly to the high volatility of the long term global bonds in its portfolio.

The building society sector had the lowest PC VaR outturn relative to all other sectors for both domestic and global bonds. This was due mainly to building societies having more instruments that were being held to maturity.

Table 7.

Comparison of Different risk Measures on Global GOJ Bonds:10-Day PC VaR versus 10-day VaR			
		10-day VaR	
	10-day Principal Component VaR	Parametric VaR	Non Parametric VaR
Commercial Banks			
CBank 1	0%	0%	0%
CBank 2	0%	0%	0%
CBank 3	0%	0%	0%
CBank 4	-7.0%	-3.9%	-5.2%
CBank 5	-7.6%	-4.2%	-5.6%
CBank 6	0%	0%	0%
CBank 7	-4.3%	-2.4%	-3.2%
Merchant Banks			
Mbank 1	-21.7%	-26.3%	-59.7%
Mbank 2	0%	0%	0%
Mbank 3	-2.4%	-1.3%	-1.8%
Building Societies			
Bsoc 1	0%	0%	0%
Bsoc 2	-3.8%	-2.1%	-2.8%
Bsoc 3	0%	0%	0%
Bsoc 4	0%	0%	0%
Mean	-3.4%	-2.9%	-5.6%

The overall VaR outturn indicates that the exposure of local banks to risk is greater for their holdings of local bonds than for their holdings of global bonds due mainly to larger concentration of domestic GOJ bonds in their portfolios (see Table 1). It can also be deduced that the composition of the bonds portfolios for the merchant banking sectors, which were typically a bullet-profile, contributed to the high PC VaR results relative to the standard 10-VaR results (see Table 2).

5. Discussion

Duration results indicate that with the evolution of market conditions, the sensitivity of global bonds to changes in the yield curve was much higher relative to domestic bonds due to the larger impact of risk factors on global bonds. The convexity outturn for global bonds was also higher relative to domestic bonds highlighting the overall high sensitivity of global bond prices to changes in market conditions.

The characteristics of duration calculations and use of PCD modeling in VaR analysis have some major advantages. PCD modeling incorporates the sensitivity of bonds to movements within the market and combines this with the decomposing characteristics of PC which evaluates risk factors individually. PCs are uncorrelated by construction and by virtue simplify VaR analysis considerably. The advantage of this form of VaR analysis increases if interest rates are assumed to not follow a multivariate normal distribution resulting in more information in the tail end of the data. The normal distribution historical VaR generates lower risk probabilities when compared to PC VaR due to the fatter tail structure of the underlying yields (see Table 6 and Table 7). Note however, in cases where an institution had a long tenor instrument that had a volatile interest rate structure the PC VaR tended to be lower than parametric and non parametric VaR results. This was evident for merchant bank 1.

The modeling of PC VaR is complimented by KRD whenever there is non-parallel interest rate movement. One example of this can be deduced from the negative and insignificant relationships between PC1 and term structure movements for the 6-month and 25-year bonds, respectively. These results reveal that expected inflation has a negative impact on short-term instruments and little or no impact on long-term instruments. Therefore, this type of analysis can be used to examine individual risk associated with domestic and international bond portfolios in the context of non-parallel movements in the yield curve arising from monetary policy[21].

6. Conclusion

This paper developed a VaR methodology for measuring interest rate risk on the banking sector’s holding of both GOJ local and global bonds. The method is intuitive and explains risk associated with portfolios using three factors that affect yield curves. Because the factors are orthogonal to each other, correlation and covariance between the yields does not have to be explored, simplifying the calculation of VaR for the portfolios explored.

[21] There is tendency for strong correlation between ‘surprises’ of monetary policy and the subsequent movement of the ‘slope’ component. This area of study is vital among central bankers and agents involved in and directly affected by central bank actions (Malava 2006).

PCD and the VaR that is generated from the model allows for the assessment of individual risk factors. This allows risk managers to isolate factors affecting term structures.

Results indicate that the risk involved with holding GOJ domestic bonds is greater than holding global GOJ bonds. This is contingent on the banking system having a large share of its portfolio bond investments in domestic bonds. Also expected inflation has a large impact of the overall risk, particularly its impact on US dollar denominated portfolios. Results of this paper indicate that the PC VaR outturn for the Jamaican banking system is higher relative to both parametric and historical VaR outturns suggesting that the PC VaR holds more information as it relates to the risks impacting banking system portfolios.

The paper recommends that the BOJ incorporates the use of PCA VaR modeling technique in monitoring risk associated with interest rate movements and its impact on banking system stability. The movements in expected inflation, market-perceived long term inflation as well as volatility of interest rates can be individually examined by regulators through the use of PCA VaR. This will allow for greater understanding by regulators of perceptions held by market participants as well as being able to assess the quantitative impact of changes in the yield curve on the portfolio risk exposures of the banking sector.

In terms of future research, the employment of Monte Carlo techniques will allow for a more detailed assessment of movement in risk factors given different scenarios. This enhancement would also allow for stress-testing exercises to be conducted on the banking sector's investments portfolio. The analysis will also be extended to capture all investment instruments held in the banking sector investment portfolio.

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Appendix 1:

Summary Statistics

GOJ Domestic par Yield Curve-Descriptive Statistics (April 1 2008 to March 18 2009)

	0.5 yr	2 yr	3 yr	6 yr	9 yr	15 yr	20 yr	25 yr
Mean	0.0019	0.0023	0.0024	0.0024	0.0023	0.0007	0.0006	0.0023
Median	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maximum	0.0523	0.3561	0.3708	0.4226	0.4215	0.4258	0.4335	0.4000
Minimum	-0.0314	-0.2188	-0.1089	-0.4571	-0.4429	-0.4294	-0.4274	-0.2079
Std. Dev.	0.0091	0.0346	0.0282	0.0498	0.0489	0.0399	0.0417	0.0449
Skewness	1.5072	4.1194	9.0189	1.5190	1.8814	-0.1383	0.2058	3.5308
Kurtosis	10.6597	57.1076	118.8034	67.7900	69.1866	106.1529	91.5266	36.8863
Jarque-Bera	711.4539	31452.8500	144225.7000	44173.1800	46145.6000	111726.3000	82289.8900	12580.5500
Augmented Dickey-Fuller	-8.228609	-16.61035	-17.91529	-14.92002	-21.80277	-14.87754	-15.8462	-22.85541
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sum	0.4762	0.5771	0.6021	0.6070	0.5726	0.1800	0.1531	0.5695
Sum Sq. Dev.	0.0208	0.3011	0.1997	0.6232	0.5995	0.3987	0.4355	0.5066
Observations	252.0000	252.0000	252.0000	252.0000	252.0000	252.0000	252.0000	252.0000

***Dickey-Fuller unit root test: 5% critical Value is equal to -2.87**

GOJ Global par Yield Curve-Descriptive Statistics (February 24 2006 to March 18 2009)

	7 yr	9 yr	20 yr	30 yr
Mean	0.000537	0.000527	0.0006	0.000595
Median	0	0	0	0
Maximum	0.159839	0.435113	0.211864	0.329919
Minimum	-0.190533	-0.192492	-0.202918	-0.324603
Std. Dev.	0.017502	0.024475	0.019449	0.029207
Skewness	0.569684	6.860503	0.912079	1.162565
Kurtosis	45.39222	144.4259	75.55154	80.85242
Jarque-Bera	59571.95	668779	174471.1	200949.7
Augmented Dickey-Fuller	-4.544742	-5.844139	-21.60681	-4.304693
Probability	0	0	0	0
Sum	0.427145	0.419288	0.476863	0.472645
Sum Sq. Dev.	0.24321	0.475622	0.300342	0.677316
Observations	795	795	795	795

***Dickey-Fuller unit root test: 5% critical Value is equal to-2.86**

Principal Component Descriptive Statistics

	PC1	PC2	PC3
Mean	0.024567	0.005104	-0.001705
Median	0.033331	-0.000279	-0.004123
Maximum	0.04255	0.042972	0.021245
Minimum	-5.30E-05	-0.00224	-0.019544
Std. Dev.	0.02027	0.015394	0.014755
Skewness	-0.32898	2.213869	0.528331
Kurtosis	1.256893	6.005444	2.014265
Jarque-Bera	1.157111	9.545854	0.69607
Augmented Dickey-Fuller*	-2.259007	2.279217	-3.243857
Probability	0.560708	0.008456	0.706074
Sum	0.196532	0.040829	-0.013637
Sum Sq. Dev.	0.002876	0.001659	0.001524
Observations	8	8	8

*** Dickey-Fuller unit root test: 5% critical Value is equal to-3.5 not however observations less than 20 makes test inaccurate**

Appendix 2

Properties of Eigenvalues and Eigenvectors

Since Q is symmetric positive semi-definite, the eigenvalues $l_j \geq 0, j = 1,2,...,n$ and the eigenvectors U_j can be orthonormalized. This relationship is easily rewritten in matrix form:

$$QV = U\Lambda \qquad [1]$$

Where $$\Lambda = \begin{matrix} l_1 & 0 & ... & 0 \\ 0 & l_2 & & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & l_n \end{matrix}$$

The matrix U consists of the orthonormal eigenvectors U_j; that is, $UU^T = U^T U = I$

Therefore, equation 1 implies

$$Q = U\Lambda U^T \qquad [2]$$

Consider the linear transformation

$$x = U\sqrt{\Lambda h} \quad [3]$$

Where $\sqrt{\Lambda} = \begin{matrix} \sqrt{l_1} & 0 & \dots & 0 \\ 0 & \sqrt{l_2} & & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \sqrt{l_n} \end{matrix}$

If $h = (h_1, h_2, ..., h_n)$ is a random vector with independent, normally-distributed components (and therefore, with the unit covariance matrix, I), then the vector, x, is a normal random vector that has the covariance matrix

$$Q_x = (U\sqrt{\Lambda})I(U\sqrt{\Lambda})^T = U\Lambda U^T = Q$$

Equation 3 can be written as

$$x = h\sqrt{l_1}U_1 + h_2\sqrt{l_2}U_2 + ... + h_n\sqrt{l_n}U_n \quad [4]$$

The random variables $z_j = \sqrt{l_j h_j}, j = 1,....,n$ are called the principal components of the random variable x, the vector $U_j, j = 1,...,n$ is referred to as the direction of the j-th principal component z_j and equation 4 is the principal component expansion of x.

Equation 4 is essential in Principal Component Analysis. Based on equations, scenarios using the vector of independent standard normal random variables $h = (h_1, h_2, ..., h_n)$ were generated. If the last several eigenvalues $l_{k+l}, ..., l_n$ are small, then the truncated vector

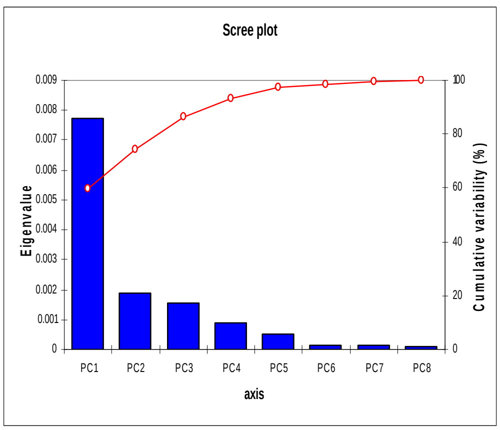
$$\hat{x} = h\sqrt{l_1}U_1 + h_2\sqrt{l_2}U_2 + \ldots + h_n\sqrt{l_n}U_n \qquad [5]$$

that uses only the vector $\hat{h} = (h_1, h_2, \ldots, h_n)$ will be a good approximation of the random vector x .

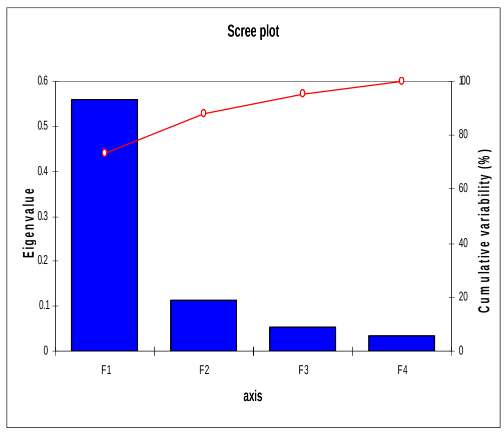
Appendix 3

Screen Plots for PC: Shows cumulative variance explained by first three PC

Domestic Bonds



Global Bonds



Appendix 4

Matlab Code for Computing PCA

```
[M,N] = size(data);
% subtract off the mean for each dimension
mn = mean(data,2);
data = data - repmat(mn,1,N);

% calculate the covariance matrix
covariance = 1 / (N-1) * data * data';

% find the eigenvectors and eigenvalues
[PC, V] = eig(covariance);
```

```
% extract diagonal of matrix as vector
V = diag(V);

% sort the variances in decreasing order
[junk, rindices] = sort(-1*V);
V = V(rindices);
PC = PC(:,rindices);
```