



# **Maximum Likelihood Approach To The Estimation of Markov Transition Matrices Using Proportions Data: An Application to Credit Risk**

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## **Abstract**

Estimates of transition matrices are critical inputs for credit risk modelling as they are used as basic building blocks in credit risk assessment. Using a variant of maximum likelihood estimation technique, this paper utilizes non-performing loans data to provide estimates of transition probability matrices for the banking sector of Jamaica. This method is effective in assessing changes in credit quality in the absence of an institution-specific credit rating. The findings of this paper suggest that the credit risk management within the banking sector is backward looking. The paper proposes a more forward looking approach to credit risk management in order to reduce credit risk exposures and increase the profitability of the banking sector. The paper postulates that by creating incentives for a more forward looking credit risk assessment framework, the banking system could realize a reduction in credit risk exposure and an increase in the banking sector's profitability as a result of reducing specific sectoral collateral requirements.

**Keywords:** Markov transition matrix; credit risk; non-performing loans; maximum likelihood

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# 1. Introduction

Credit ratings provide information on the current credit standing of obligors while rating migration patterns indicate how credit standings may change over the modelling horizon.<sup>2</sup> Transition probability matrices estimates are critical inputs for credit risk modelling as basic building blocks in credit risk assessment. Further, transition probability estimates facilitate the quantification of the economic value of movement between credit classes. Using a variant of maximum likelihood estimation technique, this paper utilizes non-performing loans data to provide estimates of transition probability matrices for the credit portfolio of the banking sector in Jamaica.

The estimation of transition probability matrices in the Jamaican banking sector improve the overall credit risk assessment from a regulatory stand point as it enhances regulators' ability to predict the performance of the banking sector's loan portfolios. Transition probability estimates enable regulatory authorities to anticipate and mitigate sharp deteriorations in the quality of institutions' loan portfolios based on anticipated macro-economic conditions. Further, when incorporated in credit risk models, transition probability estimates are useful in signalling to regulators the necessary adjustments to loan provision requirements. More importantly, transition probability estimates allow regulators to make loan provision adjustments to specific loan buckets. As such, accurate and efficient estimates of transition probabilities between credit qualities are paramount in ensuring and maintaining the credit quality within the loan book of the banking sector in Jamaica.

This paper makes use of the basic framework highlighted in Jones' (2005) paper, which was an improvement on the seminal paper of Lee et al. (1970) and incorporates the use of a variant of the Maximum Likelihood in the estimation process. Specifically, this paper integrates asymptotic efficiency gains from maximum likelihood estimation with efficiency gains from linear programming estimation in the assessment of migration

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<sup>2</sup> Jackson et al., June 1999. "Credit risk modeling," Financial Stability Review: June 1999, Regulatory Policy Division, Bank of England.

between credit classes.<sup>3</sup> The remaining sections of this paper are structured as follows; Section 2 briefly looks at the various approaches used to estimate credit transition probabilities, Section 3 gives a brief description of the data used, Section 4 outlines the methodology and Section 5 presents the estimation results. The paper concludes with Section 6 and suggests possible ways to improve the overall estimation process.

## 2. Literature Review

There has been a plethora of studies done on the estimation of Markov transition matrices with special application to credit risk. Most of the recent studies reflect improvements to an original study done by Lee et al. (1970), which imposes a first-order stationary Markov process in the estimation of credit transition matrices. Lee et al. (1970) assumed that transition between credit quality could be explained by the linear relationships between aggregate proportions,  $y_j(t)$  and  $y_i(t-1)$  which represented the proportion of observations with credit quality  $j$  and  $i$ , respectively. The estimated occurrence of  $y_j(t)$  is represented in **equation 1** below:

$$y_j(t) = \sum_i y_i(t-1)p_{ij} + u_j(t) \quad (1)$$

The above specification yielded the least square estimators. Previously, Judge and Takayama (1966) implemented restrictions on  $p_{ij}$  as used by Telser (1963) with additional measures to correct for the presence of heteroscedasticity in Equation (1). As a caveat, Lee et al. (1970) suggested the use of weighted inequality restricted least squares estimators to correct for the heteroscedasticity inherent in the formulation of the problem.<sup>4,5</sup>

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<sup>3</sup> From a statistical point of view, MLEs and linear programming estimates are considered very robust and efficient methods for using the observed behaviour of the data to formulate curvature information and thus iterates to the true value in the most efficient number of iterations.

<sup>4</sup> Heteroscedasticity is inherently present as the variance of the error term,  $u_j(t)$ , depends upon the size of  $y_i(t-1)$ .

<sup>5</sup> MacRae C. Elizabeth, 1977. "Estimation of Time-Varying Markov Processes with Aggregate Data," *Econometrica*, Vol. 45, No.1. (January 1977) pp. 190.

Alternatively, MacRae (1977) proposed that researchers could correct for heteroscedasticity through the use of Generalized Least Squares in which the covariance matrix is replaced by a consistent estimate of the covariance matrix. The process used by MacRae was called Iterative Generalized Least Squares.<sup>6</sup>

Jones (2005) outlined the use of a variant of Generalized Least Squares methodology to estimate transition probability matrices for the commercial banks in the United States using both non-performing loans data and interest coverage data. The transition probability matrices were then used to ascertain the impact of economic performance on credit quality by conditioning the transition probability estimates on macroeconomic variables. Jones (2005), using data collected from FDIC, employed four proportions calculated as a percentage of total loans, leases, and cumulative loans charge-offs. These proportions were performing loans and leases; loans and leases past due 30 to 89 days; loans and leases past due 90 days or more, loans and leases nonaccrual status and cumulative charge-offs on loans and leases.<sup>7</sup>

In the presence of a structural break, Jones (2005) revealed that transition probability estimates varied over the post and pre- break periods. Subsequently, the estimated transition matrices were conditioned for periods of low real GDP growth and high real GDP growth, as it was thought that the deterioration in real GDP was matched by the decline in performing loans ratio. Such an application was employed earlier by Bangia et al. (2002) where transition probability estimates were conditioned on business cycles. As evidence, Bangia et al. (2002) found that the high yield portfolios migration to low yield portfolios in periods of contraction might have been severely understated.<sup>8</sup>

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<sup>6</sup> This is similar in nature to the Aitken's Generalized Least Squares approach covered by Lee et al. (1970) and Zellner and Theil (1962).

<sup>7</sup> The alternative technique used by Jones (2005) made use of the Interest Coverage Ratio (where ICR is defined as the ratio of earnings before interest, taxes, depreciation, amortization to interest expenses) as a proxy for the underlining creditworthiness of a firm. The ICR calculated for each firm was then categorized into distinct groups and applied to the above equation (1).

<sup>8</sup> Bangia et al. estimated transition probability matrices under different framework where the changes in credit quality of a firm occurs under the premise that asset value evolves over time and that default was triggered by the drop in firm's asset value below the value of its callable liabilities.

Alternatively, Christodoulakis (2006) viewed transition probabilities as parameters from a Bayesian perspective which allowed for the imposition of non-negativity constraints to the transition probabilities using prior densities. The transition probability estimates were calculated via Monte Carlo Integration (MCI).<sup>9</sup> Christodoulakis' (2006) utilized a prior distribution combined with sampling information, as captured by the likelihood function, to provide the joint posterior density function of the model parameters.<sup>10</sup> The use of MCI allowed for the computation of posterior distribution of arbitrary functions of parameters.

Another approach to estimate transition probability matrices was to generate or simulate paths of credit using regime-shifting regressions as done by Hamilton (1989) and Bangia et al. (2002). Hamilton (1989) made use of a variant of the Kalman filter and generated a time path of an observed series which drew inferences about an unobserved state variable. However, whereas the Kalman filter is a linear algorithm for generating estimates of a continuous unobserved state vector, Hamilton's (1989) filter and smoother provided nonlinear inference about a discrete-valued unobserved state vector.<sup>11</sup> Specifically, Hamilton (1989) solved for the marginal likelihood function for the specified variable and maximized this likelihood function with respect to population parameters. These parameters and the data were used to draw the optimal statistical inference about the unobserved regimes.

### 3. Data Description

Data used in estimating the transition probability matrices covers quarterly non-performing loans for commercial banks, building societies and merchant banks. Five loan buckets were calculated as a percentage of total loans. These were performing loans

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<sup>9</sup> Judge and Takayana (1966) dealt with the issue of Inequality Restrictions in Regression Analysis with a special application to transition probability matrix.

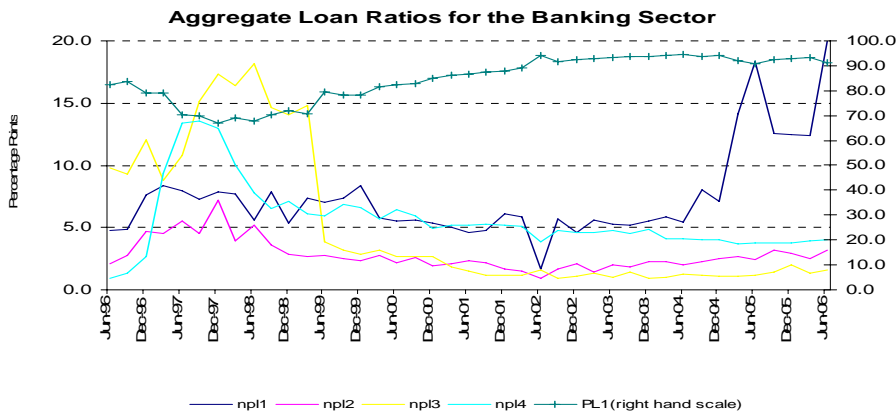
<sup>10</sup> Prior distribution is a probability distribution that quantifies knowledge regarding unknown quantities (e.g. model parameters) prior to observing the data or other information pertaining to the unknown quantities. [hea-www.harvard.edu/AstroStat/statjargon.html](http://hea-www.harvard.edu/AstroStat/statjargon.html)

<sup>11</sup> The use of a Kalman Filter in the face of non-linear function was similar in nature to the Extended Kalman Filter, which was linearized about the current mean and covariance.

(np1), loans past due 30-90 days (np11), loans past due 91-180 days (np12), loans past due 181-365 days (np13) and loans past due over 365 days (np14).

The macro-economic variables used in the regression analysis were real Gross Domestic Product (GDP), real weighted average loan rate, real weighted average selling exchange rate (JAD/USD), unemployment rate and inflation rate. These macro-economic variables were thought to have the greatest *a priori* deterministic power in explaining the changes in the various loans buckets.

**Figure 1.** Aggregate Loans Ratios for the Banking Sector

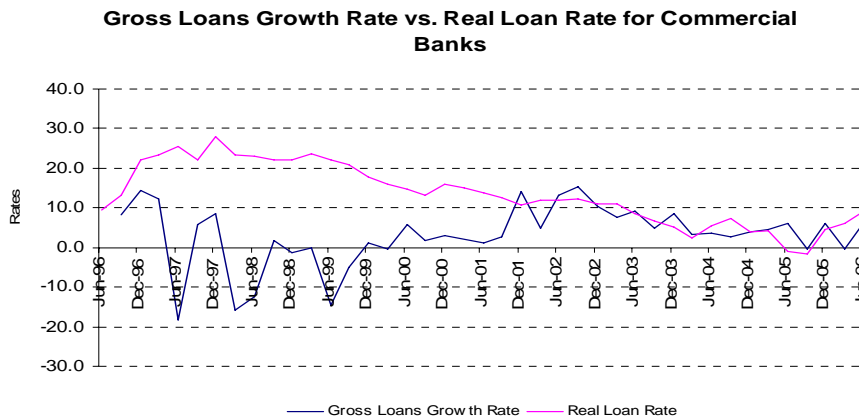


The general improvement in the credit quality of loans portfolios of the banking sector is reflected in the downward trend in non-performing loan ratios over the sample period June 1996 to June 2006 (see **Figure 1**). As a caveat, loans past due 30-90 days (np11) increased in the latter half of the specified period. This increase in aggregate loans past due 30-90 days (np11) was driven by the commercial bank and the merchant bank sectors. The highly competitive loans market coupled with the reduction in real loan rate led to the increase drive by commercial banks and merchant banks to issue new loans in an attempt to grow their loans portfolios. This loan strategy has led to the weakening of the credit administrative standards.

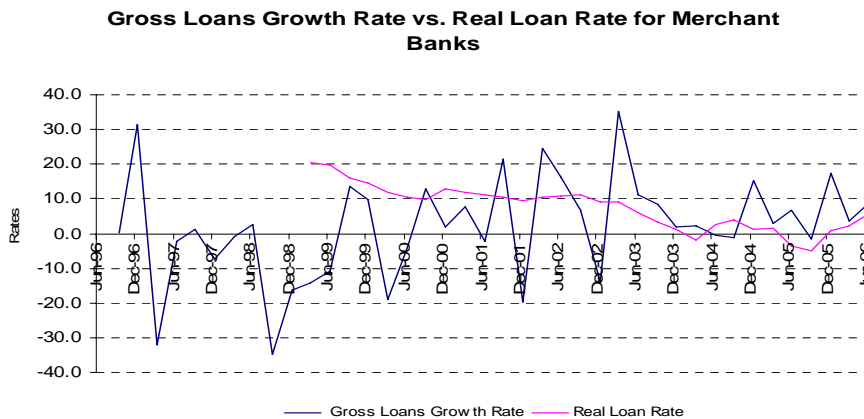
Immediately following the financial crisis of the late 1990s, the banking sector improved its' risk management infrastructure. Prudent risk management coupled with an increase in

the available credit to the public sector in an attempt to finance the large fiscal deficit led to the downward trend in real interest rates.<sup>12</sup> Further, there were positive growth in loans post December 1999, tied with the downward trend in real interest rates, suggests that the period January 2000 to June 1996 was categorized by buoyant demand for loans (see Figures 2-4).

**Figure 2.** Loan Growth Rates vs. Real Loan Rates for Commercial Banks

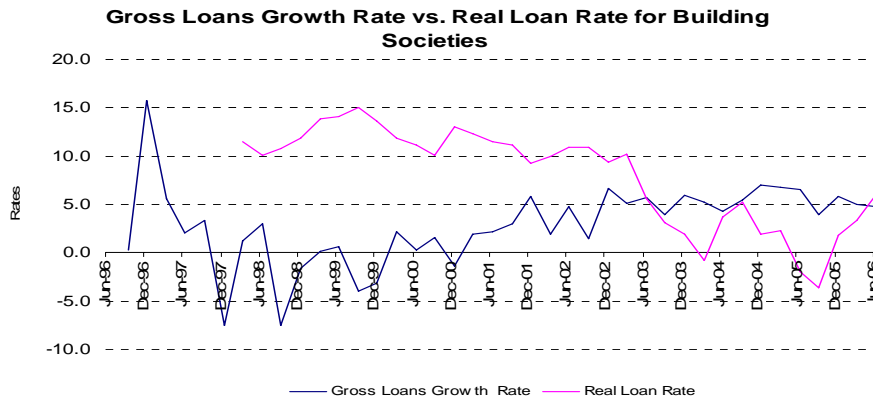


**Figure 3.** Loan Growth Rates vs. Real Loan Rates for Merchant Banks



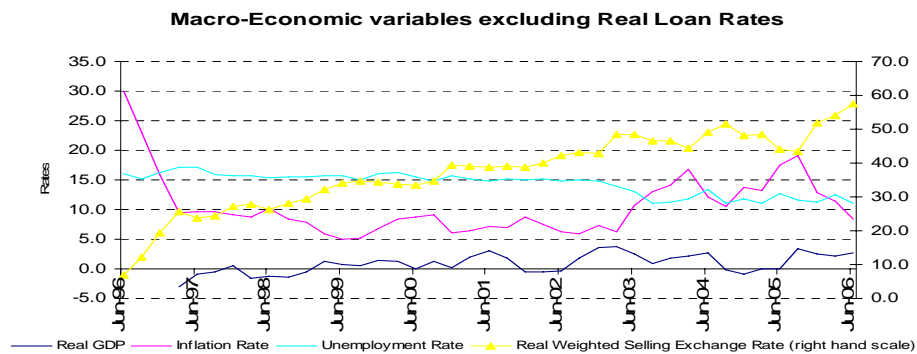
<sup>12</sup> There was also an imposition of restrictive and cautious lending to the general public during the restructuring period.

**Figure 4.** Loan Growth Rates vs. Real Loan Rates for Building Societies



There was some amount of volatility in the inflation rate and the real weighted selling exchange rate over the sample period. The volatility in the exchange rate was largely due to speculation stemming from external shocks such as rising oil prices and the terrorists attacks on the United States on 11 September 2001. Alternatively, the volatility in the inflation rate was influenced by the shocks to output of local agriculture crops as result of the occurrence of three major hurricanes and rising oil prices. (See Figure 5).

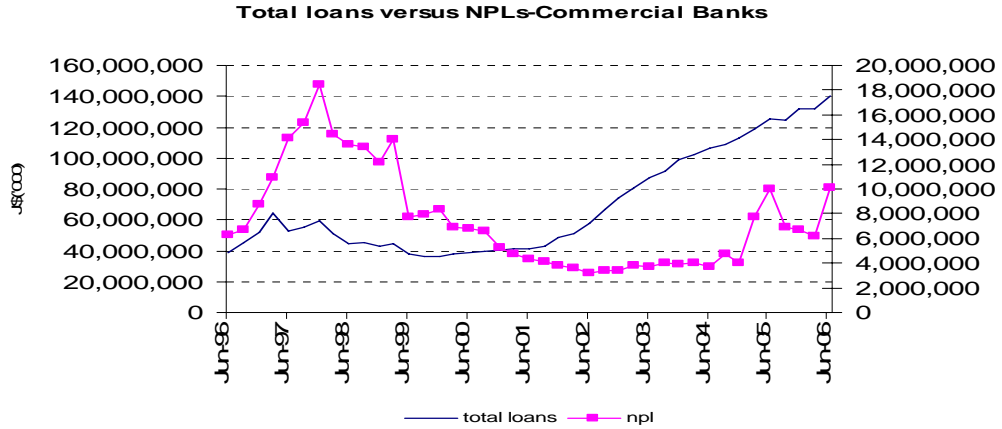
**Figure 5.** Macro-Economic Variables excluding Real Loan Rates



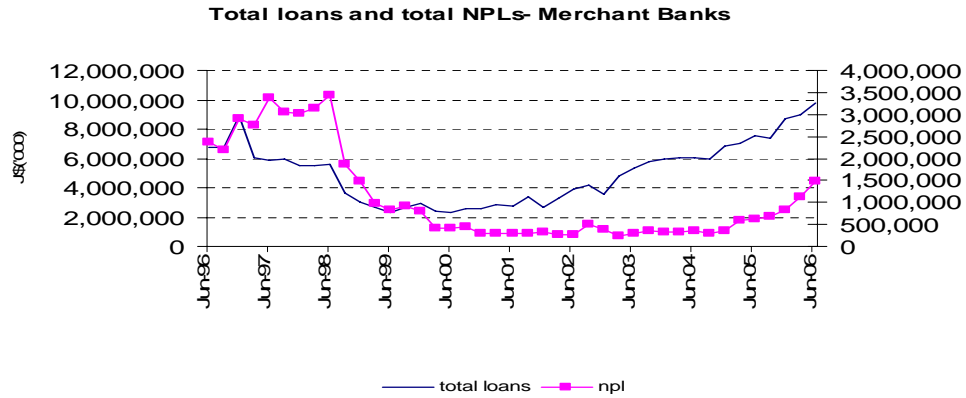
Source: Bank of Jamaica, STATIN.



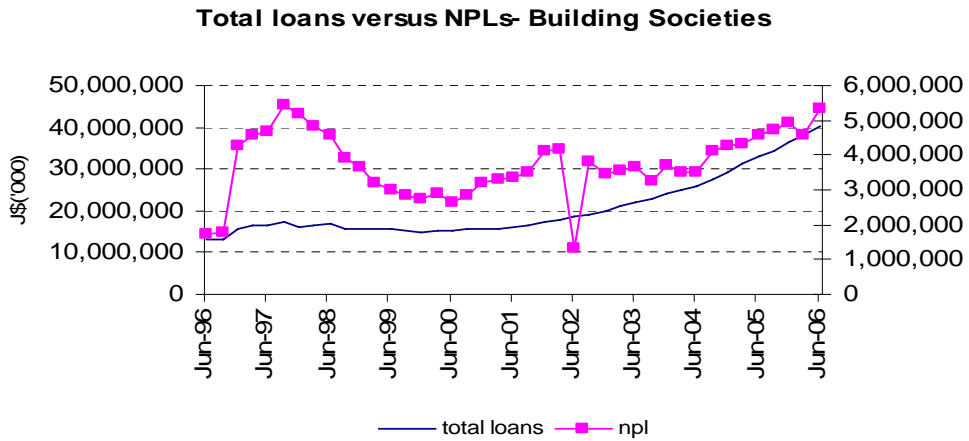
**Figure 6.** Total Loans versus Total Non-Performing Loans for the Commercial Banks



**Figure 7.** Total Loans versus Total Non-Performing Loans for the Merchant Banks



**Figure 8.** Total Loans versus Total Non-Performing Loans for Building Societies



The general decrease in the ratios of non-performing loans to total loans for the banking sector is as a result of the growth in new loans out pacing the growth in non-performing loans (See Figure 6-8).

## 4. Methodology

The use of a simple time-homogeneous Markov model implies stationary transition probabilities over the sample period considered.<sup>13,14</sup> The variant Maximum Likelihood Estimation (MLE) approach used in this paper places restrictions on the parameters of the model to ensure the transition probability estimates maintained proper probability characteristics.<sup>15</sup> MLEs are most appropriate for large samples due to their asymptotic properties. These properties include consistency, asymptotic normality, asymptotic efficiency and invariance.<sup>16</sup> As the sample size increases, MLEs out perform all other estimators as these estimators reflect the true population value.<sup>17</sup> The model equations are as follows:

$$\ln L(p_{ij}, \delta^2) = -\frac{n}{2}[\ln(2\pi) + \ln \delta^2] - \frac{1}{2} \sum_{i=1}^n \left[ \frac{y_j(t) - \sum_i y_i(t-1)p_{ij} - crisis}{\delta^2} \right] \quad (3)$$

Subject to

$$p_{ij} \geq 0 \quad (4)$$

$$\sum p_{ij} = 1 \quad (5)$$

Where *crisis* is a dummy variable used to represent the crisis period of the financial system. The crisis period covers June 1996 to December 2000.

<sup>13</sup> Keifer M. Nicholas and C.Erik Larson, 2004. "Testing Simple Markov Structures for Credit Rating Transitions," Office of the Comptroller of the Currency or the Department of the Treasury.

<sup>14</sup> Jafry and Schuermann (2004) showed that relaxing the time-homogeneity assumption has little impact on the transition probability estimates.

<sup>15</sup> Maximum Likelihood Estimation is a popular statistical method used to make inferences about parameters of the underlying probability distribution from a given data set.

<sup>16</sup> Green H. William, 1997. "Econometric Analysis," Prentice-Hall, pp. 127.

<sup>17</sup> True population value refers to the true credit quality migration probability for the entire period including the sample period.

Equations 4 and 5 were added to the model to ensure that the transition probability estimates,  $p_{ij}$  sum to one and are all non-negative.<sup>18</sup> The transition probability estimates obtain from the above specification produces the unconditional estimates in the proceeding section.<sup>19</sup> Convergence within the MLE framework is important as it has implications for the sum of squared errors and significance levels of the parameter estimates.<sup>20</sup> Initial values were obtained using Linear Programming estimates.<sup>21</sup> Given that these initial values are predicated on the information contained within the data set, they are preferred to random uninformed guesses. As such, the initial values used in the model incorporated the true nature of the data set. Furthermore, choosing initial values close to the global maxima increases the convergence rate under the Gauss-Newton method and improves the accuracy of the point estimates.<sup>22</sup>

All loans series were standardized, as done in previous studies, to reduce volatility inherent in the data without the loss of generality. The standardization of the entire loans series implicitly assumes normal distribution among the loans series. The White's heteroscedasticity consistent estimator is used to estimate the asymptotic covariance of the parameters in the model. The use of the White's heteroscedasticity consistent estimator ensured that appropriate inferences can be made from the regressions results without knowing the precise nature of the heteroscedasticity.<sup>23</sup>

Macro-economic variables were included as additional explanatory variables to ascertain their impact on the estimated transition probabilities. In particular, Real GDP growth was computed as a binary series which assigns "1" for the occurrence of the current quarter's annualized growth rate being greater than the previous quarter's annualized

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<sup>18</sup> These restrictions ensured the properties of probability estimates were not violated.

<sup>19</sup> These are transition probability estimates excluding macro-economic variables.

<sup>20</sup> Convergence within the MLE approach makes use of the Gauss-Newton method. The Gauss-Newton method starts by computing the associated change in sum of error squares with every change in the value of each parameter. This procedure will inform about the slope of the sum of error squares surface at the initial values.

<sup>21</sup> Kiountouzis (1973) outlines the some properties of the Linear Programming technique. For example, LP estimates are comparable to Least Squares estimates under the assumption of normal distribution.

<sup>22</sup> Baker A. David, 2006. "Maximum Likelihood Estimate of Parameters," <http://www.umiacs.umd.edu/research/EXPAR/papers/ITIP-543/node7.html> January 8, 2007.

<sup>23</sup> Green H. William, 1997. "Econometric Analysis," Prentice-Hall, pp. 463.

growth rate and “0” otherwise. The associated change to the transition probability estimates as a result of adding these macro-economic variables feeds through the model via the computed change in sum of error squares with every change in the value of each parameter. As stated previously, this is largely explicative of the Gauss-Newton method in the Maximum Likelihood approach. If the initial starting values are close enough to the true maximum, then the Gauss-Newton method usually converges to global maxima, typically with a linear convergence rate.

The model with the macro-economic variables is as follows:

$$\ln L(p_{ij}, \delta^2) = -\frac{n}{2} [\ln(2\pi) + \ln \delta^2] - \frac{1}{2} \sum_{i=1}^n \left[ \frac{y_j(t) - \sum_i y_i(t-1)p_{ij} - crisis - \sum_{i=1}^n x_i}{\delta^2} \right] \quad (6)$$

Subject to equations (4) and (5).

Where  $x_i$  represents the macro-economic variables included in the model. The transition probability estimates derived from the above specification produces the conditional estimates.<sup>24</sup>

Tests for structural breaks within the various series were applied to each series. Additionally, unit root tests were examined to ascertain whether the data was stationary. Both tests for structural breaks and unit roots made use of the Phillips-Perron technique.

## 5. Transition Probability Estimates<sup>25</sup>

Economic theory suggests that the leading diagonal of transitional probability matrices should account for the largest portion of the probability estimates. Therefore, from

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<sup>24</sup> Conditional transition probability estimates simply means that the transition probability estimates are re-estimated with the inclusion of macro-economic variables as specified.

<sup>25</sup> The validation for the use of macro-economic variables lagged one period versus several lags can be found in appendices.

economic theory, the highest probability of movement should be for the various credit classes to remain in their respective credit classes. In cases where the largest probability estimates are to the right of the leading diagonal, this implies a general deterioration of the credit qualities.

### i. Unconditional Transition Probability Estimates

The estimated transition probabilities for the banking sector derived from the MLE approach excluding the macro-economic variables are shown below.

**Table 1.** Estimated Quarterly Transition Matrix for Jamaica Commercial Banks Loans  
(All Commercial Banks, 1996:2-2006:2)

		To				
		PL1	NPL1	NPL2	NPL3	NPL4
From	PL1	0.399	0.072	0.054	0.474	0.000
	NPL1	0.000	1.000	0.000	0.000	0.000
	NPL2	0.171	0.068	0.402	0.023	0.336
	NPL3	0.552	0.000	0.000	0.316	0.133
	NPL4	0.179	0.000	0.084	0.000	0.737

**Table 2.** Estimated Quarterly Transition Matrix for Jamaica Merchant Banks Loans  
(All Merchant Banks, 1996:2-2006:2)

		To				
		PL1	NPL1	NPL2	NPL3	NPL4
From	PL1	0.425	0.000	0.023	0.183	0.369
	NPL1	0.000	1.000	0.000	0.000	0.000
	NPL2	0.074	0.287	0.571	0.068	0.000
	NPL3	0.550	0.000	0.281	0.169	0.000
	NPL4	0.020	0.281	0.386	0.000	0.313

**Table 3.** Estimated Quarterly Transition Matrix for Jamaica Building Societies Loans  
(All Building Societies, 1996:2-2006:2)

		To				
		PL1	NPL1	NPL2	NPL3	NPL4
From	PL1	0.893	0.000	0.063	0.000	0.043
	NPL1	0.000	1.000	0.000	0.000	0.000
	NPL2	0.065	0.470	0.133	0.000	0.332
	NPL3	0.099	0.000	0.229	0.618	0.054
	NPL4	0.043	0.000	0.360	0.000	0.597

The unconditional transitional probability estimates of commercial banks and merchant banks indicate that approximately **55.0 per cent** of loans past due over 181-365 days will become performing over a given quarter. Approximately **39.9 per cent** of performing loans within the commercial banks' loans portfolios will remain performing, while **47.4 per cent** of these performing loans will be past due 181-365 days. Similarly, **42.5 per cent** of performing loans within the merchant banks loans portfolios will remain performing, while **36.9 per cent** of these performing loans will be past due over 365 days. The building societies sector, which primarily extends mortgage loans, had the most robust credit risk profile with **89.3 per cent** of performing loans likely to remain performing within the next quarter. However, it is the general view that the transitions between credit qualities are influenced by macro-economic conditions. As such, these results must be qualified by quantifying the impact of these macro-economic variables on these transition probability estimates. The next section provides such an analysis.

## **ii. Conditional Transitional Probability Estimates**

The unconditional transition probability estimates ignores the impact of macro-economic variables. As such, producing transition probability estimates that reflect the impact of the significant underlying macro-economic variables gives the best indication of movement between credit qualities and a true reflection of the credit risk exposures of the banking sector. The macro-economic variables included in the analysis are real GDP, real loan rate, weighted average selling exchange rate and the inflation rate. Unemployment rate proved to be statistical insignificant in explaining the direct movement in the banking sector's loans portfolios. As such, the impact of unemployment rate is not presented in this paper.<sup>26</sup>

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<sup>26</sup> Results are available by author upon request.

**Table 4.** Estimated Quarterly Transition Matrix for Jamaica Commercial Banks Loans with the Inclusion of all significant Macro-Economic variables.

(All Commercial Banks, 1996:2-2006:2)

		To				
		PL1	NPL1	NPL2	NPL3	NPL4
From	PL1	0.347	0.000	0.175	0.478	0.000
	NPL1	0.000	1.000	0.000	0.000	0.000
	NPL2	0.454	0.066	0.000	0.117	0.363
	NPL3	0.471	0.000	0.140	0.149	0.240
	NPL4	0.434	0.000	0.000	0.000	0.566

**Table 5.** Estimated Quarterly Transition Matrix for Jamaica Merchant Banks Loans with the Inclusion of all significant Macro-Economic variables.

(All Merchant Banks, 1996:2-2006:2)

		To				
		PL1	NPL1	NPL2	NPL3	NPL4
From	PL1	0.387	0.000	0.305	0.036	0.272
	NPL1	0.000	1.000	0.000	0.000	0.000
	NPL2	0.520	0.200	0.224	0.015	0.041
	NPL3	0.769	0.000	0.231	0.000	0.000
	NPL4	0.000	0.000	0.145	0.090	0.764

**Table 6.** Estimated Quarterly Transition Matrix for Jamaica Building Societies Loans with the Inclusion of all significant Macro-Economic variables.

(All Building Societies, 1996:2-2006:2)

		To				
		PL1	NPL1	NPL2	NPL3	NPL4
From	PL1	0.877	0.000	0.050	0.072	0.000
	NPL1	0.000	1.000	0.000	0.000	0.000
	NPL2	0.101	0.240	0.000	0.000	0.660
	NPL3	0.251	0.044	0.135	0.307	0.263
	NPL4	0.000	0.048	0.163	0.134	0.655

The transition probability estimates of commercial banks and the merchant banks are indicative of the backward looking credit risk management system within the banking sector. This assertion is evidenced by the high probability of non-performing loans becoming performing loans (p11). For commercial banks, **47.8 per cent** of performing loans are expected to be past due 181-365 days on the one hand, while **47.1 per cent** of

loans past due 181-365 days (npl3) is expected to be performing, on the other hand. Similarly for merchant banks **30.5 per cent** of performing loans (pl1) is expected to be past due 91-180 days, while **52.0 per cent** and **76.9 per cent** of loans past due 91-180 days and 181-365 days (npl2 & npl3) are expected to become performing (see **Tables 4-6**).<sup>27</sup> With the expected *net improvements* in non-performing loans buckets, the current full collateralized borrowing needs to be adjusted to reflect these net improvements.

The net movement between performing loans (pl1) and loans past due 181-365 days (npl3) for the financial system accounts for the institutions' attempt to avoid being penalized by the Bank in the form of full disclosure of loan terms and structure for loans past due over 181 days. Loans past due over 181 days are carefully monitored by the Bank's on-site inspectors. Therefore, institutions will increase their collection efforts on outstanding amounts to avoid being warned by on-site inspectors and avoid the possibility of having to write off these accounts as bad debts as well as the additional request to increase capital to mitigate these exposures. Although these results signal the effectiveness of the Bank's on-site supervision, they are also indicative of a backward-looking credit risk management system within the Jamaican banking sector.

A backward-looking credit risk management system reflects the nature of these institutions to place greater emphasis on *collecting* on loans past due rather than ensuring the issue and monitoring of high quality loans. More detailed evaluation of the net movement between performing loans (pl1) and loans past due (npl3) could be explained by the sector profile of performing and non-performing loans issued by the banks. The sectoral break out of the non-performing loans on a quarterly basis is paramount for the type of analysis done here.<sup>28</sup>

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<sup>27</sup> The building societies largely reflected economic theory with the highest probability estimates along the leading diagonal with the only exception being loans past due 91- 180 days. Only 7.2 per cent of performing loans is expected to be past due 181-365 days, while 25.1 per cent of loans past due 181-365 days is expected to move to the performing loans bucket (pl1).

<sup>28</sup> This data is currently not available.



There is a seeming paradox with the co-existence of a general improvement in the credit quality of the loans within the commercial banks and merchant banks loans portfolios and the high probability of performing loans becoming non-performing within 181-365 days over the period. The key to understanding their coexistence is to focus on the *net flows* between credit classes. That is, the high probability of performing loans becoming non-performing within 181-365 days is more than counterbalanced by a higher probability of non-performing loans becoming performing on aggregate over the same period. This analysis, however, reveals the ‘Achilles’ heel’ of a ‘backward looking’ credit risk management system. That is, large deteriorations in credit quality are possible with existing levels of performing loans becoming non-performing and significant fall out in non-performing loans becoming performing. Such an eventuality is likely in the event of a sectoral shock which hampers capacity to repay outstanding loans amounts *as well as* loan payments which are currently due.

The general movement between credit classes indicates that the present collateral requirements could be lowered with a more forward looking credit risk management as suggested by the estimated net improvement in the credit risk profile of the banking sector loans portfolios.

## **6. Discussion and Conclusion**

In this paper, estimates for the first-order stationary Markov process are developed. The analysis of the banking sector reveals estimates largely reflective of economic theory. As such, underpin the idea that the changes in credit quality of loans portfolios are largely dependent on the movements in the macro-economic variables. Real GDP growth, weighted selling exchange rate, loan rates and the inflation rate are statistically significant in explaining direct changes in the various loans categories. It should be noted, these variables are influenced by the growth in monetary base.<sup>29</sup> As such, the Bank can influence the banking system’s credit risk exposures and institution specific credit ratings by targeting the growth in monetary base.

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<sup>29</sup> The equation for the Quantity theory of money explains the relationship with the related variables and the growth in money ( i.e.  $MV=PY$ ).

In the last five years increases in real GDP growth and the general downward trend in real loan rate coupled with the high demand for loans has encouraged the migration towards performing loans category. However, this has been counter-balanced by the deterioration of the credit worthiness of the banking sector's loans portfolios due to increases in real weighted selling exchange rate. This paper postulates that exogenous variables that were correlated with the considered macro-economic variables are major contributors to the observed changes in credit quality of the various banking sector loans portfolios. These exogenous variables could include, but are not limited to, demand for loans, growth in the informal sector and gains from foreign currency investments.

The results highlight the need for a more forward looking approach to credit risk management in an effort to mitigate credit risk. Further, the results suggest that with proper forward looking credit risk management system, collateral requirements to specific sectors (e.g. for loans to the tourism sector) could be lowered. This assertion is supported by the fact that there is a large amount of non-performing loans becoming performing as the banking sector attempt to collect on loans past due over 181 days in an attempt to avoid being censored by the Bank. Lowering the collateral requirements, in a context of a better and improved forward looking credit risk management within banks would enable the banking sector to increase its supply of high quality loans at a faster rate to meet the increase demand for loanable funds and thus augment the net interest income margin of the banking system.

Armed with the vital and critical basic building blocks to properly and effectively conduct credit risk assessment of the banking sector of Jamaica, the Bank could develop effective policies with these directives at the center of its policy plans. The proposed policy directives are to institute sectoral and institution-specific collateral requirements based on the credit quality of institution's loans portfolio as indicated by the transition probability matrices. The Bank would also encourage, monitor and reward banks with improved credit administrative system with lower provisioning requirements (on the part of the bank) as well as lower required collateral (on the part of their potential borrowers).

The current credit risk management framework does not allow for institution-specific collateral requirements. As such, the findings of this paper imply that the use of the transition probability matrices when augmented with the existing supervisory framework will allow the Bank to effect the necessary adjustments to sectoral and institution-specific collateral requirements. These adjustments will depend on the expected change in credit quality of the various loans portfolios of the banking sector. However, sectoral breakouts of the various non-performing loans buckets in quarterly frequency would facilitate the assessment of the sectoral specific credit risk exposures and hence facilitate targeted provisioning and collateral requirements.

## **Appendices**

The Maximum Likelihood Estimators follow a chi-squared distribution with degrees of freedom equal to number of data points minus number of fitted parameters. The Maximum values from the Maximum Likelihood Estimation when compared to chi-squared statistic at the specific degrees of freedom is considered as a measure of goodness-of-fit. With this in mind, the following tests below were carried out with the intention to prove which model was best suited given for the maximum likelihood estimation given the number of available observations.

Given the sample size (June 1996 to June 2006) the model with the inclusion the macro-economic variables lagged one period were better suited for the data set when compared with the model with macro-economic variables lagged several periods. Such a conclusion was derived by the fact that in all cases both variations of the model displayed similar measure of goodness-of-fit and therefore with a small sample macro-economic variables lagged one period was chosen. In addition, LRT revealed that the inclusion of the macro-economic variables current and lagged two period though true in many cases, does not improve the overall goodness-of-fit. Such a conclusion was evidenced by the rejection of the inclusion of the macro-economic variables lagged two periods and the current period's values as being statistically different from the model with macro-economic variables lagged one period in cases where they both failed to measure the data set well.(**See Appendix A-C**)

# Appendix A

## Commercial Banks

Goodness-of-fit-WEXRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	bad	bad
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

Goodness-of-fit-UNEMPLOY		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	bad	bad
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

Goodness-of-fit-RLRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	bad	bad
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

Goodness-of-fit-IRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	bad	bad
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

## Likelihood Ratio Test (LRT)

Likelihood Ratio Test-WEXRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	TRUE	9.046854 (0.01085177)
NPL1	TRUE	365.836129 (0.000000)
NPL2	NOT TRUE	4.705329 (0.09511539)
NPL3	NOT TRUE	0.715562 (0.69922628)
NPL4	TRUE	20.316245 (0.00003876)

Likelihood Ratio Test-UNEMPLOY		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	NOT TRUE	1.257463 (0.53326792)
NPL1	NOT TRUE	3.800530 (0.14952895)
NPL2	NOT TRUE	3.440413 (0.17902917)
NPL3	NOT TRUE	0.141019 (0.931918869)
NPL4	TRUE	5.476230 (0.06469217)

Likelihood Ratio Test-RLRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	NOT TRUE	0.574001 (0.75051131)
NPL1	TRUE	15.206152 (0.00049891)
NPL2	TRUE	32.755605 (0.0000008)
NPL3	NOT TRUE	0.381727 (0.82624542)
NPL4	NOT TRUE	4.993008 (0.08237247)

Likelihood Ratio Test-IRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	NOT TRUE	2.544092 (0.28025759)
NPL1	TRUE	15.083133 (0.00053057)
NPL2	NOT TRUE	2.488057 (0.2882082)
NPL3	NOT TRUE	1.309379 (0.51960341)
NPL4	TRUE	13.757465 (0.00102945)

## Appendix B

### Merchant Banks

Goodness-of-fit-WEXRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	good	good
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

Goodness-of-fit-UNEMPLOY		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	good	good
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

Goodness-of-fit-RLRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	bad	bad
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	bad	bad

Goodness-of-fit-IRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	good	good
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

## Likelihood Ratio Test (LRT)

Likelihood Ratio Test-WEXRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	NOT TRUE	2.593330 (0.27344215)
NPL1	TRUE	370.130656 (0.0000)
NPL2	NOT TRUE	0.639166 (0.72645187)
NPL3	NOT TRUE	0.276740 (0.87077628)
NPL4	NOT TRUE	1.702717 (0.42683466)

Likelihood Ratio Test-UNEMPLOY		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	NOT TRUE	1.604729 (0.44826778)
NPL1	NOT TRUE	0.398951 (0.81916014)
NPL2	NOT TRUE	2.512770 (0.28468130)
NPL3	NOT TRUE	4.716187 (0.0946004)
NPL4	NOT TRUE	0.905359 (0.63592195)

Likelihood Ratio Test-RLRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	NOT TRUE	3.398087 (0.18285838)
NPL1	NOT TRUE	1.037383 (0.59529889)
NPL2	NOT TRUE	3.025931 (0.22025588)
NPL3	NOT TRUE	1.880757 (0.39048003)
NPL4	NOT TRUE	4.576511 (0.10144326)

Likelihood Ratio Test-IRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	NOT TRUE	0.992615 (0.60877430)
NPL1	TRUE	362.720263 (0.0000)
NPL2	NOT TRUE	2.240105 (0.3262661)
NPL3	NOT TRUE	0.416622 (0.81195429)
NPL4	NOT TRUE	1.111888 (0.57353071)



## Appendix C

### Building Societies

Goodness-of-fit-WEXRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	good	good
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

Goodness-of-fit-UNEMPLOY		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	good	good
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

Goodness-of-fit-RLRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	good	good
NPL1	bad	bad
NPL2	good	good
NPL3	bad	bad
NPL4	bad	bad

Goodness-of-fit-IRATE		
Dependent Variables	Macro-Variables with 1 lag	Macro-Variables with current and 1-2 lags
PL1	good	good
NPL1	bad	bad
NPL2	good	good
NPL3	good	good
NPL4	good	good

## Likelihood Ratio Test (LRT)

Likelihood Ratio Test-WEXRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	TRUE	25.389795 (0.00000307)
NPL1	TRUE	10.985498 (0.00411651)
NPL2	NOT TRUE	5.394665 (0.06738502)
NPL3	TRUE	7.583912 (0.02255145)
NPL4	TRUE	12.052862 (0.00241409)

Likelihood Ratio Test-UNEMPLOY		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	TRUE	13.428090 (0.00121374)
NPL1	TRUE	115.617536 (0.0000)
NPL2	NOT TRUE	0.390291 (0.82271495)
NPL3	TRUE	10.483874 (0.00529000)
NPL4	TRUE	15.984736 (0.00033803)

Likelihood Ratio Test-RLRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	TRUE	137.2000438 (0.0000)
NPL1	TRUE	12.565932 (0.00186785)
NPL2	NOT TRUE	1.677720 (0.43220293)
NPL3	TRUE	20.743607 (0.00003130)
NPL4	NOT TRUE	1.483911 (0.47618175)

Likelihood Ratio Test-IRATE		
Dependent Variables	Restriction(i.e. inclusion of current period's value and lag 2 period)	Chi-squared(2) & (significance level)
PL1	TRUE	25.711276 (0.00000261)
NPL1	TRUE	6.648354 (0.03600214)
NPL2	TRUE	7.924725 (0.01901813)
NPL3	NOT TRUE	2.875098 (0.23750914)
NPL4	TRUE	7.277227 (0.02628877)

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