Application of the Government of Jamaica Zero-Coupon Curve to Modelling Yield Curve Risk

Oma Coke*

Financial Stability Department

December 8, 2016

Abstract

This study uses the Svensson (1994) method to estimate quarterly Government of Jamaica (GOJ) Zero-Coupon yield curves from March 2014 to March 2016. The Svensson (1994) method of estimation was used to obtain the parsimonious yield curve. The estimated spot rate curve is then incorporated into an interest rates stress testing framework to assess the impact on portfolio holdings of parallel and non-parallel shifts of the yield curve. The results of the stress testing exercise show that exposure to parallel shifts of the curve were higher across the respective market participant groups relative to non-parallel shifts.

Additionally, DTIs and securities dealers were more vulnerable to shifts in medium term segment of the yield curve. The life insurance sub-sector was more vulnerable to the long end of the yield curve while the general insurance sub-sector exposures were equally weighted across the short to medium term segment of the curve.

JEL Classification: F31, F32, F41

Keywords: Yield Curve, Key Rate Duration, Financial Stability

*The views of this study does not necessarily reflect the views of the Bank of Jamaica.

Contents

1 Introduction

The yield curve depicts the relationship between bond yields against their maturity. It can be used as a benchmark for pricing bonds and in value analysis more generally. In practice, the estimation of a yield curve is often derived from observations of market prices in the government debt market. The use of the government's debt portfolio may be attributable to the fact that in most jurisdictions the government is the largest issuer of bond; coupled with the perceived risk profile - theoretically risk free and practically low risk. The yield curve is also a useful indicator for central banks as they are able to capture changes in market expectations of macroeconomic conditions, monetary policy and investors risk preferences.

In light of the aforementioned, this study addressed two objectives. Firstly, a yield curve for the period 2014Q1 to 2016Q1 is estimated using Government of Jamaica (GOJ) domestic issued JMD denominated bonds. To accomplish this objective, the study used the Svensson (1994) parametric model to infer GOJ's yield curve from domestic bond prices. The choice of Svensson model was motivated by the increased flexibility of the curve while maintaining the parametric properties of the curve that provides sound economic intuition. The estimation of GOJ yield curve is motivated by Kladivko (2010) who uses the Nelson-Siegel model for Czech Treasury yield curve from 1999 to the present and Gurkaynak, Sack, and Wright (2006) who use the Svensson model to estimate the U.S. Treasury curve from 1961 to the present. Further motivation for this paper was garnered from Langrin (2007) who estimated multi-factor versions of the Vasicek (1977) and the Cox, Ingersoll and Ross (CIR; 1985) models of term structure of interest rates for GOJ zero-coupon bond prices. The estimation by Langrin (2007) was conducted via state space modelling on daily GOJ domestic bond yields from 24 September 2004 to 28 July 2006 obtained from Bloomberg. Unlike Langrin (2007), which relies on an affine diffusion term structure modelling, this study relies on a cross-sectional approach to estimate the GOJ domestic zero-coupon yield curve.

Secondly, since interest rate risk can be captured by changes in the yield curve, this study considers estimation of the key rate durations of the GOJ's domestic bond portfolio. The study further assesses the impact of shifts in the yield curve guided by the key rate duration model on portfolio holding of domestic issues by market participant groups.

This approach adds to the existing work of Tracey (2009) who employs principal component analysis and key rate durations for assessing interest rate risk of holdings of both local and global GOJ bonds by Jamaica's banking sector.

This study is organized as follows: section 2 reviews the fundamental concepts of the yield curve; section 3 presents the Svensson modelling framework; section 4 provides an overview of the data used in model including a detailed discussion of inherent issues; section 5 presents the results of the estimation, including an assessment of the fit of the curve; section 6 demonstrates the application of the key rate duration model in assessing the impact of yield curve shifts on portfolio holdings of JMD denominated domestic government issues for existing market participant groups in Jamaica's financial system; and section 7 concludes.

2 Yield Curve Basics

This section provides a review of the fundamental concepts of bond pricing and the development of a yield curve.

2.1 The Discount Function and Zero-Coupon Yields

The pricing of a bond is conditional on the present value of its future cash flows. The interest rate or discount function used to calculate the present value depends on the yield offered on comparable securities in the market. The discount function is used to maintain real value across the time, i.e., time value of money. In theory, the application of the discount function to value a zero-coupon bond that pays $1 in n years can be written as:

$$P_t = \delta_t(n) = e^{-r_t(n)} \times n, \tag{1}$$

where $\delta_t(n)$ denotes the continuous discount function as at time t and $r_t(n)$ is the continuously compounded rate of return (yield) demanded by the investor for holding such investment until n periods ahead of time t (n denotes the time to maturity). The subscript t denotes the variability of the discount function. From equation (1) above, one may apply the necessary transposition to get an expression for the continuously compounded yield (spot rate) on the zero-coupon bond:

$$r_t(n) = \frac{-ln(\delta_t(n))}{n}. \tag{2}$$

In applying the concept of compounding to bond pricing, one may consider expressing yields on a coupon-equivalent basis. In this case the compounding may be assumed to be m times per year instead of being continuous (e.g. semi-annual compounding implies that $m = 2$, the payment of coupon is 2 times per year). Thus we express the relationship between the continuously compounded yield and the m-compounded coupon-equivalent as

$$r_t(n) = m \times ln(1 + \frac{r_t^{ce}(n)}{m}) \tag{3}$$

where $\frac{r_t^{ce}(n)}{m}$ denotes the coupon-equivalent yield compounded m times per year. Similarly, the discount function is expressed as

$$\delta_t(n) = \frac{1}{(1 + \frac{r_t^{ce}(n)}{m})^{m \times n}}. \tag{4}$$

Thus the relation between yields and coupon equivalent yields creates ease of mobility between continuously compounding and its coupon equivalent counter parts. The relationship between yields and maturities are captured by the yield curve.

2.2 Coupon Bond and the Par Yield Curve

Similar to zero-coupon bonds, the pricing of a coupon bearing bond is conditional on the discount function; thus the price is the sum of the discounted future cash flows of the bond. For illustration, consider the price of a coupon-bearing bond with nominal value of 100 and coupon payment of \$C (C= $\frac{100c}{m}$) that matures in exactly n years from time t as follows:

$$P_t(n) = \sum_{i=1}^{m \times n} C\delta_t(i/m) + 100\delta_t(n), \tag{5}$$

where $\delta_t(i), i = 1, 2, ..., n$ are discount functions for their respective maturities. Note that the yield on a coupon-bearing bond is dependent on the coupon rate that is assumed. One implication of this condition, as pointed out by Gurkaynak et al., (2006), is the disparity in the yields of bonds with identical maturities but varying coupon values.

The yields on a coupon bearing bond can be expressed in terms of par yields. A par yield may be defined as the coupon rate at which a bond with a specific maturity would be traded at par, that is, the rate at which the present value of the bond is equivalent to its nominal value. Hence, given a coupon bearing bond with a nominal value of \$100 and maturity n, the par yield is obtain as follows:

$$100 = \frac{100c_t(n)}{m}\sum_{i=1}^{m\times n}\delta_t(i/m) + 100\delta_t(n), \tag{6}$$

where $c_t(n)$ denotes the n year par yield. From the equation above the par yield can be expressed as

$$c_t(n) = \frac{m(1-\delta_t(n))}{\sum_{i=1}^{m\times n}\delta_t(i/m)}. \tag{7}$$

The par yield serves as a proxy for the quotation of yield on a coupon bearing bond by financial market participants (Gurkaynak, Sack and Wright, 2006). As discussed, the yield curve, once estimated, may be presented as a zero coupon yield curve or a par yield curve. The curvature of the yield curve will reflect the sensitivity of bond prices to interest rates and is measured by the bonds duration and convexity.

2.3 Duration and Convexity

The duration of a bond is a measure of the sensitivity of a bond's value to changes in interest rates. This measure, modified duration, can easily be derived from the Macaulay duration methodology. Frederick Macaulay (1938) defines duration (coined as the Macaulay duration) on coupon-bearing bond as the weighted average of the time (in years) that the investor must wait to receive their cash flows , that can be a expressed as:

$$D = \frac{1}{P_t(n)}\left(\sum_{i=1}^{m\times n}\frac{i}{m}\frac{c}{m}\delta_t(i/m) + n\delta_t(n)\right) \tag{8}$$

where $\frac{c}{m}$ denotes the annual coupon payment compounded m times per year for a bond instrument. Bonds that pay coupon has a duration that is less than its maturity while for the case of a zero-coupon bond, its duration is equal to its maturity. From equation (8) it is observed that for constant maturity and spot rate, the modified duration is inversely related to the coupon rate, i.e. higher coupon rate results in shorter duration for a given maturity. In the context of application, the modified duration is mostly consider. Unlike the Macaulay duration, the modified duration primarily assumes that the expected cash flow of the bond does not change when the yield changes.

The modified duration can be defined in terms of the Macaulay duration as the duration of the bond divided by one plus the yield on the bond (for a selected compounded period):

$$D^M = \frac{D}{(1 + \frac{r_t^{ce}}{m})}. \tag{9}$$

Duration in general captures a linear relationship between price changes and yield change. Thus the measure is accurate for changes in bond price relative to small changes in yield. The nonlinearity of the relationship between bond prices and yield to maturity impedes on the accuracy of the duration measure to capture effective price changes relative to large changes in yield. The nonlinear relationship between price and yield to maturity is effectively accounted for in the measure of convexity. So in a simplistic point of view, convexity is used to measure the portion of the bond price change relative to the change in the yield to maturity that is not accounted for in the duration measure. This can be depicted through the second-order Taylor approximation of bond price changes with respect to yield,

$$\frac{\Delta P_t(n)}{P_t(n)} \approx -D^M \Delta y_t + \frac{1}{2} C (\Delta y_t)^2 \tag{10}$$

where $C = \frac{1}{P_t(n)} \frac{d^2 P_t(n)}{d y_t^2}$ is the convexity of the bond. Convexity accounts for the uncertainty in yields observed at the long end of the yield curve which results in the yield curve depicting a concave shape. An implication of this is that the capital gain from a decline in the yield is higher than the capital loss from an increase in the yield. Notably, bonds with longer maturity portraying higher convexity results at times in what is referred to as convexity bias. The greater the convexity bias is, the more concave the yield curve will become. More details of the impact of convexity on the functional form of the yield curve are provided below.

3 Model Selection and Overview

The modelling of a yield curve can be broadly categorized into two groups: 1) parsimonious models and 2) spline-based models (see Waggoner, 1994). Between the two groups one has to decide on their preference in regard to the trade-off between accuracy which is an advantage of the latter and smoothness which is an advantage of the prior.

The Bank for International Settlements (BIS, 2005) reports that nine out of thirteen central banks which report their yield curve estimates to the BIS use the parsimonious approach. The popularity of parsimonious models among central banks may be attributed to the inherent property of the parsimonious approach in providing sufficiently smooth yield curves which are consistent with underlying macroeconomic conditions and investors' preferences. Spline-based methods on the other hand provide a richer precision in the fitting of the curve and is a preferred choice if one is interest in small pricing anomalies. However, spline-based yield curves may not be smooth enough and may oscillate considerably over daily intervals (Kladivko, 2010).

In this paper the parsimonious approach to estimating the yield curve for Jamaica was adopted. Under this framework, the Nelson-Siegel (Nelson and Siegel, 1987) and Svensson (Svensson, 1994) models are presented throughout the remainder of this section.

In their seminal work on yield curves, Nelson and Siegel (1987) assumed that the functional form for the instantaneous forward rate is the solution of a second-order differential equation whose roots are equal:

$$f(\tau) = \beta_0 + \beta_1 e^{-\lambda\tau} + \beta_2 \lambda\tau e^{-\lambda\tau} \tag{11}$$

where $f(\tau)$ is the instantaneous forward rate for the τ periods ahead, $\theta = (\beta_0, \beta_1, \beta_2, \lambda)$ is a vector of parameters to be estimated. equation (11) may be classified as a three component exponential function. The first component β_0 is known as the level and may be defined as the limit of the forward rate as τ tends to infinity (i.e. the asymptotic rate at which the forward rate and spot rate converges).The second component, $\beta_1 e^{-\lambda\tau}$, controls the slope of the forward rate curve and is a monotonically decreasing term (if β_1 is positive) or increasing term (if β_1 is negative). The third component, $\beta_2 \lambda e^{-\lambda\tau}$ controls the location and size of the hump in the forward rate curve (β_2 determines the magnitude and sign of the hump while λ determines the location of the hump).

Integrating equation (11) (with respect to τ) from 0 to τ and dividing the outcome by τ we get the continuously compounded spot rate curve:

$$i_c(\tau) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \tag{12}$$

where the subscript c denotes continuity. From equation (12), one can compute the corresponding discount function by applying the established relationship:

$$\delta(\tau) = e^{-i_c(\tau)\tau}. \tag{13}$$

The discount function can be used to price outstanding issue with specific coupon rate and maturity dates. The asymptotic properties of the model provides rich economic intuition. The curve (forward or spot) by definition converges to finite limits from both ends. Note that:

$$\lim_{\tau \to \infty} f(\tau) \equiv \lim_{\tau \to \infty} i_c(\tau) = \beta_0 \tag{14}$$

and

$$\lim_{\tau \to 0} f(\tau) \equiv \lim_{\tau \to 0} i_c(\tau) = \beta_0 + \beta_1. \tag{15}$$

From the above limits, we observe that the instantaneous forward and spot rates can be approximated as the sum of the β_0 and β_1 while β_0 is an approximation of the long-run rate (aka the steady-state level). Fitting the long-end of the term structure of the yield curve may be difficult as the convexity effects on bonds tends to pull down the yields on longer maturities (Gurkaynak et al. 2006). Gurkaynak et al. (2006) highlighted that the Nelson-Siegel specification tends to have forward rates asymptote too quickly to be able to capture the convexity effects at longer maturities.

The Nelson-Siegel model was later extended by Svensson (1995) through the inclusion of an additional exponential term which accounts for a second hump in the forward rate curve. The inclusion of this term increase the flexibility of the curve and improved the data fit. The functional form of the forward rate curve specified by Svensson (1995) is:

$$f(\tau) = \beta_0 + \beta_1 e^{-\lambda\tau} + \beta_2 \lambda\tau e^{-\lambda\tau} + \beta_3 \gamma\tau e^{-\gamma\tau} \tag{16}$$

where $\theta = (\beta_0, \beta_1, \beta_2, \beta_3, \lambda, \gamma)$ is a vector of parameters to be estimated. Similarly, the location and size of the second hump is governed by $\beta_3 and \gamma$. Note that the Svensson model collapses to a Nelson-Siegel model if $\beta_3 = 0$. Integrating equation (16) (with respect to τ) from 0 to τ and dividing the result by τ resulted in the continuously compounded spot rate curve:

$$\begin{aligned} i_c(\tau) =& \beta_0 + \beta_1\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \\ &+ \beta_3\left(\frac{1-e^{-\gamma\tau}}{\gamma\tau} - e^{-\gamma\tau}\right) \end{aligned} \tag{17}$$

Similar to the Nelson-Siegel model, the Svensson model converges to similar limiting points at both ends of the curve. The estimation of the Svensson model relies on fitting data to equation (16) to obtain the beta coefficients, λ and γ parameters.

4 Data and Estimation Issues

4.1 Method of Estimation

In estimating the yield curve, the Svensson method was considered. The procedural method of estimation adopted in the study follows closely to that of *Kladivko (*2010*)*.[1] The estimation of the parameters relies on the minimization of the weighted sum of squared deviations between the actual and predicted bond prices of coupon bonds:

$$\hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{N} \left(\frac{P_i - \hat{P}_i}{P_i D_i^M} \right)^2 \tag{18}$$

where N is the number of observed bonds, P_i is the observed dirty price of the coupon bond, θ is the vector of parameters to be estimated, $\hat{P}_i$ is the estimated bond price which is obtain from the model spot rates, equation (1) the discount function and equation (4) the bond price formula. Similar to *Kladivko, (*2010*)*, the inverse of the product of observed bond prices and modified duration, $(1/P_i D_i^M)$ were adopted as the optimization weight. The continuously compounded spot rates were obtained under the day count convention of Actual/360 for interest accrued.

The implementation of equation (16) was conducted with *Lsqnonlin* in MatLab, a nonlinear least squares algorithm developed in Coleman and Li (1996). Due to its flexibility, *Lsqnonlin* allows for setting of the lower and upper bound of parameter(s) to be optimized, hence making it ideal for estimating parametric models of the yield curve. However, a drawback of the optimization algorithm *Lsqnonlin* is its sensitivity to the initial value of λ, (Kladivko, 2010) . Kladivko, (2010) advised that given the true value of λ, the algorithm converges robustly to the true values of β parameters of the parametric model of interest. From this he concludes that the *Lsqnonlin* algorithm succeeds in finding the global minima. Dispite these pros and cons, the initialization of the parameters of the models follows closely to that of Kladivko, (2010) and Gurkaynak et al. (2006).

[1] The MatLab codes developed by Kladivko (2010)were utilized forthis paper.

The estimation of parameters of the yield curve may suffer from abrupt changes in their values from one period to the next. Such changes were referred to as catastrophic jumps by Cairns and Pritchard (2001). In addressing catastrophic jumps in the estimated level component of the yield curve, β_0, Kladivko (2010) imposes a lower bound on the possible values that λ and γ may assume. Additionally, Kladivko (2010) restricted β_0 to be positive which is in line with the theory. These constraints give rise to restrictions on the parametric models as pointed out by Kladivko (2010). Kladivko (2010) further pointed out in his study that the restricted Nelson Siegel model does not perform much differently when compared to the unrestricted Nelson Siegel model. However, unlike Kladivko (2010) who relies on daily data for his analysis, this study utilizes quarterly data on bond prices which makes it difficult to observe catastrophic jumps in the parameter estimates. mean absoulte

4.2 Data Set

The study utilizes quarterly market values of domestic GOJ bonds reported by domestic market participants for the period 2014Q1 to 2016Q1. This sample period was chosen because the data that were available prior to the selected period were perceived to be noisy in relation to the developments that took place in 2010 and 2012. During the first quarter of 2010, the GOJ conducted a restructuring of their debt portfolio. The restructuring of the government's portfolio was due primarily to the chaellenge in servicing the existing debts at the given maturities. As such there was a shift in most maturities to longer tenor. Similar actions were performed by the government in the first quarter of 2012. Since then, the government has reduced its participation in the domestic market significantly.

To date, the existing domestic bond market lags behind that of developed and transitional states as trades in these instruments are not captured in a formal trading system. In light of this, the market value reported by the domestic participants at the end of the quarters were used to extract the average bond prices. The data used in the study came from two primary sources: Financial Services Commission (FSC) for information on nonbank financial institutions and Bank of Jamaica (BOJ) for information on deposit taking institutions.

In improving the quality of the estimation, a data filtering process was developed. For the period under study, the following data cleaning was conducted:

i Benchmark Investment Notes identified by the GOJ were utilized.[2]

ii Floating interest rate bonds were excluded since their use in estimating the yield curve is not straight forward.

iii For each benchmark notes, bond prices that exceed two standard deviations about its mean were excluded from the analysis so as to minimize possible distortions in the data.

iv No adjustments for tax or coupon effects were made.

v Bonds that were issued for more than one year and mature within six months are excluded as they distort the liquidity conditions in the market.

vi Bonds that were issued for less than six months that matures over one year were also excluded from the sample due to their liquidity conditions.

In total, 12 GOJ bonds data were used for the period under study. In fitting the short end of the curve, the one month, three months and six months Treasury bill rates were utilized. The fitting of the short end reduces the likelihood of obtaining negative rates or extremely high rates which is important in the estimation process. A key advantage of the data reported is the richness of information collected.

5 Estimation Results

Using the above methodology, the Svensson yield curve was estimated for the period March 2014 to March 2016. The evolution of the estimated curve throughout the period was fairly stable as observed from the parameter estimates (see Figure 1).[3] The level parameter of the model fluctuated around a marginally improving trend within the bands of 8 and 19 percent. With the exception of the third quarter 2014, the slope parameter of the model posited a slightly upward trend below the zero mark. Similarly, the curvature parameters (i.e. λ and γ) were slightly trending upward over the sample period. The interest rate spread between the 10 year and 1 year yields gently sloped

[2] Includes domestic issued JMD denominated securities that have a noncallable feature.

[3] It was noted throughout the sample period that there were quarters in which the estimated results of Svensson model imply over parameterization (see appendix A1). Alternatively, one may estimate a Nelson-Siegel model which was also considered by the study.

upwards over the estimation horizon. At the long-end, the spread between 35 year and 10 year yields, fluctuated around a relative downward sloping trend line. The interest rate spread between the 1 year and 10 year yields was highest for 2015Q3 where the corresponding spread at the long end of the curve was lowered.[4] This outturn to some extent reflected investors' preference along the maturity spectrum for the GOJ's domestic JMD issues. At the long end of the curve, interest rate spread was highest for 2015Q1 which corresponded to a decrease in the corresponding interest rate spread for the 1 to 10 years yields when compared to 2014Q4.[5] For the period 2014Q4, interest rate spreads for 1 to 10 years yields and 10 to 30 years yields recorded positive quarterly growth, thus reflecting to some extent increased preference for higher yields across the entire maturity spectrum of the GOJ domestic JMD issue.[6] The flattening of the curve at the long end was most evident for 2014Q3 which reflected the minimum interest rate spread for 10 to 30 years yields over the sample period.

In sum, the estimated outputs throughout the sample period provided upward sloping yield curves.[7] The fit of the model to the observed sample data was most accurate as at end-2015 as displayed by the incorporated error measures.

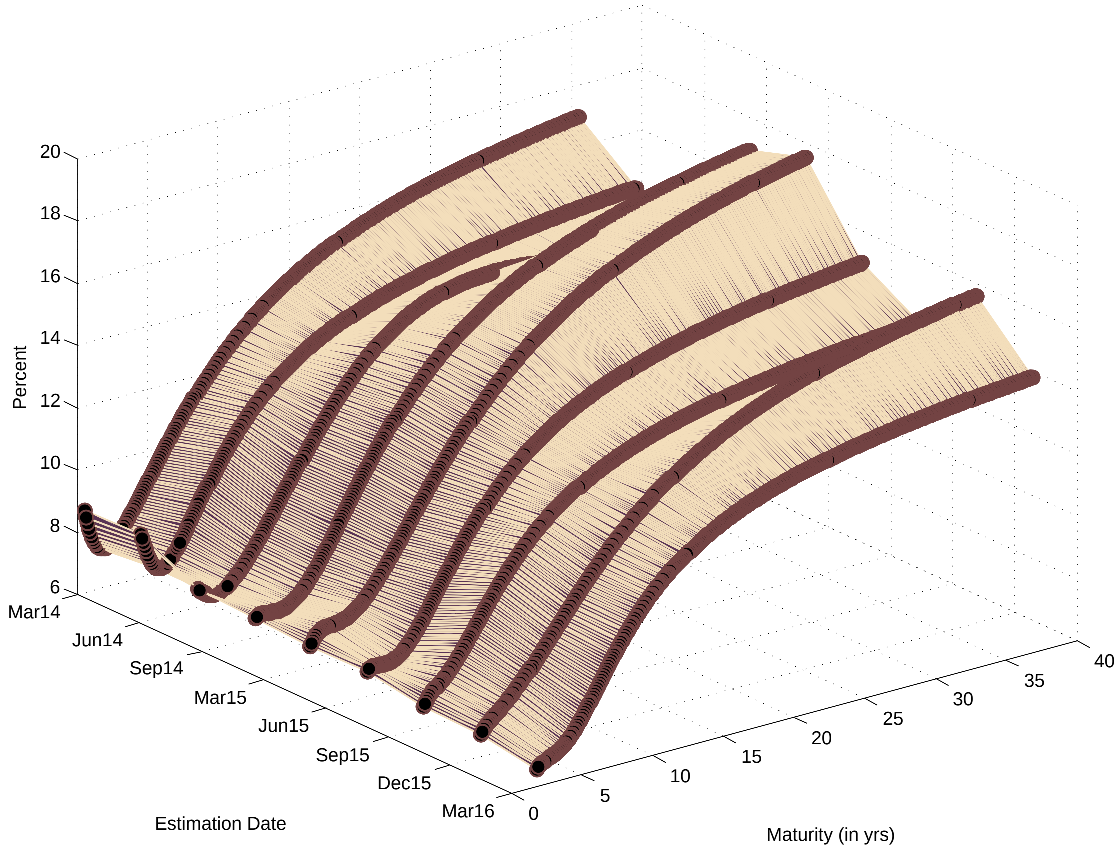
[4]The 1 to 10 years spread on yields was 4.6 percent reflecting a 10.1 percent increased relative to 2015Q2 while the 10 to 30 years yield spread was 2.6 percent reflecting a 29.5 percent decline relative to the prior quarter.

[5]The interest spread between the 10 and 30 year yields was 5.9 percent reflecting 12.7 percent increase while the 1 to 10 years interest rate spread was 4.1 percent reflecting a decline of 2.8 percent.

[6]The interest spread between the 1 to 10 years and 10 to 30 years yields were 4.2 percent and 5.3 percent reflecting quarterly increases of 5.3 percent and 178.5 percent, respectively.

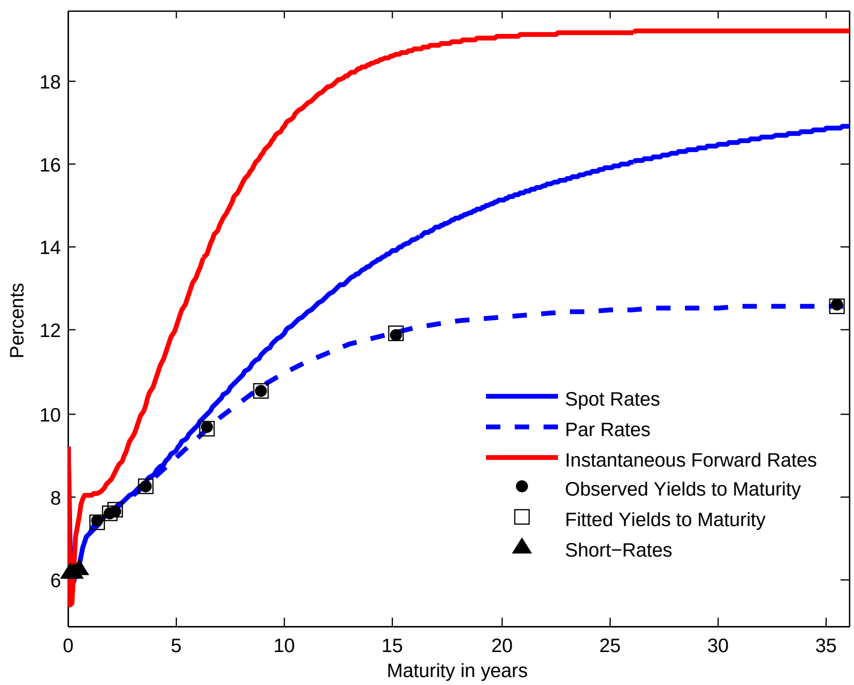
[7]see Estrella and Trubin (2006)

Figure 1: Estimated Zero-coupon yield curves (continuously compounding) for the period March 2014 to March 2016



As an example of the results, the estimated spot, instantaneous forward and par rates for December 2015 were captured by Figure 2. The rates are presented as annually compounded. There were 8 government bonds available as at end-2015 with maturities ranging from approximately one year and four months to approximately thirty five years.

Figure 2: The Svensson Model for December 2015



As can be seen from Figure 2, the Svensson curve provides a fair fit of the term structure of the government's domestic debt. However, the fit of the curve was poorer at the short-end of the curve (less than one year) reflecting the idiosyncratic nature of these issues. For the one to five years maturity bucket, the fit of the 2019 8.5 coupon bond was the worst which appeared to be overpriced relative to the other bonds. The shape of the estimated spot rate curve was upward sloping for maturities over three years. At the short end, a U-shaped hump was evident. This suggests market participants' expectation of monetary easing by the central bank in the short term, (Bomfin, 2003).

Similar to Kladivko (2010), the Mean Absoulte Error (MAE), the Root Mean Squared Error (RSME) and the Maximum Absolute Error (MaxAE) were used to assess the goodness of fit of the model:

$$RSME = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{19}$$

$$MaxAE = max_i\{|y_i - \hat{y}_i|\}, i = 1, ..., n \tag{20}$$

where n is the number of government bonds for a given settlement date, y_i is the observed yield to maturity, and $\hat{y}_i$ is the fitted yield to maturity. In calculating the error measures, the Treasury bill rates were excluded from the analysis.[8]

Table 1: Error Measures for Estimated Yield Curve

Svensson Estimated Yield to Maturity Curve as at end December 2015		
RSME (bps.)	MAE (bps.)	MaxAE (bps.)
3.8	3.3	6.7

The estimated MaxAE which identifies the point of least best fit was associated with the 2018 7.75 percent coupon bond. The MaxAE for the estimated 2015Q4 zero-coupon curve reflected the overpricing of the 2018 7.75 percent coupon bond relative to the corresponding estimated output.

6 Stress Testing Application of the Yield Curve

The yield curve has many applications that are localized to the intended purposes. For example, inflation expectation which is of critical importance for monetary policy can be obtained from the yield curve. Additionally, Estrella and Trubin (2006), investigated the use of the yield curve as a forecasting tool in real time of macroeconomic conditions. The study employed a probabilistic model to capture the relationship between key attributes of the curve (i.e. the steepness of the curve) and the business cycle, for which they found that the yield curve was a good predictor of recessions.

Seminal work of Ho (1992) utilized non-parallel shifts in the yield curve as an approach for fixed income portfolio immunization. Ho (1992) investigated the impact of changes in selected rates along the curve on the pricing of fixed income securities. This approach is currently coined *key rate duration (KRD)* and is commonly used among financial market practitioners in developing hedging strategies for their portfolio holdings.

[8]The exclusion of the error measures for Treasury bill rates was motivated by the poor fit of the curve at the short end. In addition, yields on Treasury bill were not collected in the sample.

This paper applied the key rate model to GOJ's domestic sovereign portfolio within the context of assessing interest rate risk exposure. Such applications involved shifting of the zero-coupon curve through selected key rates for the GOJ domestic JMD bond portfolio. With these key rates, one has the flexibility to conduct parallel and non-parallel shifts of the curve to provide richer analysis of bond price movements.

6.0.1 Key Rate Model

For this section, the KRD and Key Rate Convexity measures of interest rate risk are discussed along the lines of application for stress testing. The KRD as defined by Ho (1992) is a measure of the price sensitivity of a fixed income security to changes in selected spot rates along the yield curve. These rates are referred to as the key rates. Ho (1992) who pioneered the application of the KRD for fixed income portfolio recommended 11 key rates - $1, 2, 3, 4, 5, 7, 9, 10, 15, 20$ and 30 years to maturity. It is important to note that the choice of key rates along the yield curve is flexible in that one can choose any number of rates rate along the curve. The KRD measure is used by market practitioners to decompose portfolio returns, identify interest rate risk exposure, design active trading strategies and implement passive portfolio strategies such as portfolio immunization and index replication, (Nawalkha, Soto and Beliaeva, 2005).

The use of the key rate model is conditional on the assumption that any smooth change in the term structure of zero-coupon yields can be represented as a vector of changes in a number of properly chosen key rates. That is:

$$\Delta Y = (\Delta y(t_1), \Delta y(t_2), ..., \Delta y(t_m)) \tag{21}$$

where Y is the zero-coupon curve and $\Delta y(t_i)$ for $i = 1, 2, ..., m$ are the set of m key rates. Changes in all other interest rates are approximated by linear interpolation of the changes in the adjacent key rates. The shifting of a key rate along the zero-coupon curve, only impacts rates within the neighborhood of the selected key rate that are bounded to the right and the left by the closest key rates to our key rate of interest, (Nawalkha, Soto and Beliaeva, 2005). Rates outside of this bound will be unchanged. The shortest and longest key rates are bounded on one side, the shortest key rate is bounded to the right by the second key rate while the longest key rate is bounded to the left by the $m - 1st$ key rate. Thus, shifting the shortest key rate by an amount x results in an equal amount in shifting rates to the left of the shortest key rate and a linear interpolation of the shift in rates to the right of the key rates that are bounded, while leaving rates above the bound unchanged. Similarly, shifting the longest key rate results in an equal shift of rates to the right of the longest key rate and linear interpolation of the shift in rates to the left of the longest key rate that are bounded, while leaving all other rates below the bound unchanged. A generic expression for the change in the interest rate for any given term t is written as:

$$\Delta y(t) = \begin{cases} \Delta y(t_{shortest}) & t \leq t_{shortest} \\ \Delta y(t_{longest}) & t \geq t_{longest} \\ \alpha \times \Delta y(t_{left}) + (1-\alpha) \times \Delta y(t_{right}) & else \end{cases} \tag{22}$$

where $y(t_{shortest})$ and $y(t_{longest})$ are the shortest and longest key rates, $y(t_{left})$ and $y(t_{right})$, with $t_{left} \leq t \leq t_{right}$, refers to the key rate adjacent (to the left and the right) to term t, and α and $(1-\alpha)$ are the coefficients of the linear interpolation, defined as:

$$\alpha = \frac{t_{right} - t}{t_{right} - t_{left}}$$
$$1 - \alpha = \frac{t - t_{left}}{t_{right} - t_{left}}.$$

The set of key rate shifts can be used to evaluate the change in the price of fixed income securities. An infinitesimal shift in a given key rate, $\Delta y(t_i)$, results in an instantaneous price change given as:

$$\frac{\Delta P_i}{P} = -KRD_i \times \Delta y(t_i) \tag{23}$$

where KRD_i is the $i-th$ KRD. So the key rate is defined as the negative percentage change in the price of a given fixed income security resulting from the change in the $i-th$ key rate:

$$KRD_i = -\frac{1}{P}\frac{\delta P}{\delta y(t_i)}. \tag{24}$$

Alternatively, the duration of the $i-th$ key rate is defined as the negative of the elasticity of the price of a given fixed income security to the $i-th$ key rate relative to the $i-th$ key rate:

$$KRD_i = -\frac{e_{p,i}}{y(t_i)}, \tag{25}$$

where $e_{p,i}$ is the elasticity of the price to the $i-th$ key rate. The application of the key rate model is fairly straight forward. First, we calculate the KRD for each of our 5 key rates using the formula:

$$\frac{\delta P}{\delta y(t_i)} = \frac{\delta P}{\delta y(t)}\frac{\delta y(t)}{\delta y(t_i)} = -\frac{CF_t \times t}{e^{y(t)\times t}}\frac{\delta y(t)}{\delta y(t_i)}. \tag{26}$$

By substituting equation (22) into (18) we have:

$$KRD_i = t \times \frac{\delta y(t)}{\delta y(t_i)}, \tag{27}$$

where t is the time to maturity. Observe that the KRD is an increasing function of time. Thus key rates at the long end of the curve would have a greater responsiveness of price changes to interest rate changes.

The total price change resulting from all key rate changes is given as:

$$\begin{aligned} \Delta P &= \Delta P_1 + \Delta P_2 + ... + \Delta P_m \\ &= -\sum_{i=1}^{m} KRD_i \times \Delta y(t_i). \end{aligned} \tag{28}$$

The sum of the KRD measures from a simultaneous shift in all the key rates by the same amount result in the traditional duration of a given fixed income security. Thus, the KRD measure only account for the linear effect of key rate shifts. Under a non-infinitesimal shift in the term structure, the KRD framework is extended to account for second-order nonlinear effects of such shift. The nonlinear effect of the key rate shifts is called the Key Rate Convexity (KRC) and is defined as:

$$KRC(i,j) = KRC(j,i) = \frac{1}{P}\frac{\delta^2 P}{\delta y(t_i)\delta y(t_j)} \tag{29}$$

for every pair (i, j) of key rates. Similarly, the sum of the KRC measures from a simultaneous shift in all the key rates by the same amount result in the traditional convexity of a given fixed income security. The KRDs and KRCs of a portfolio can be obtained as the weighted average of the KRD and KRCs of the securities in the portfolio.

The following section discusses the selection of the key rates that will be used in our KRD model to conduct parallel and non-parallel shifts of the yield curve. Such shifts of zero-coupon curve will be governed by scenario analyses that are acceptable industry practices.

6.1 Application of the Key Rate Model

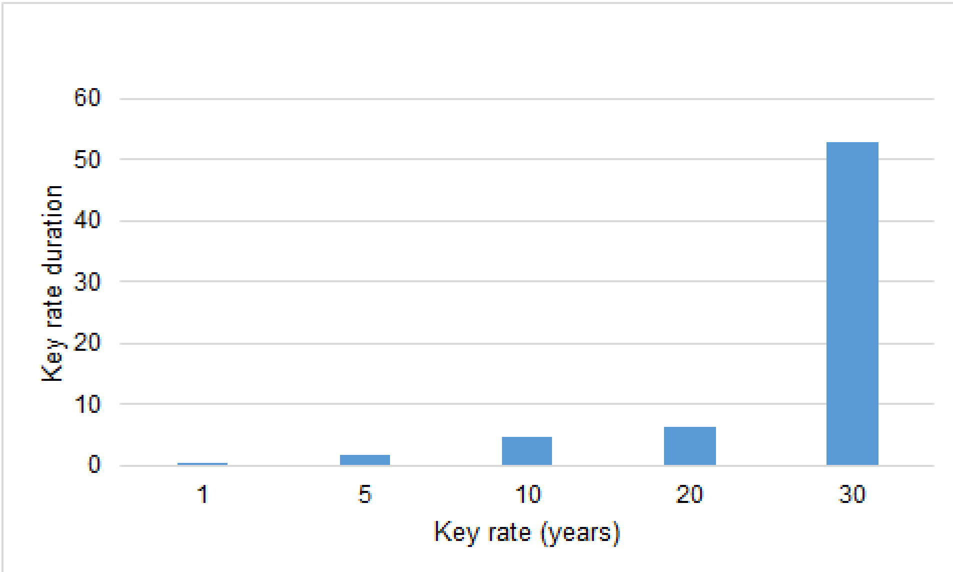
The choice of key rates as pointed out by Zeballos (2011) is arbitrary owing mainly to the absence of unique fundamentals. In acknowledgment of this gap in the model framework, Nawalkha et al. (2005) proposed that the choice of key rates can be guided by the maturity structure of the portfolio under consideration. As such, the choice of key rates for this analysis will be guided by the structure of the government's domestic fixed income portfolio.

As at end March 2016, total outstanding JMD denominated government's issue was approximately J$233 billion in nominal value for fixed coupon bonds and J$508 billion in nominal value for variable coupon bonds which is unevenly distributed across 33 issues. This outstanding debt issue is sparsely distributed across the maturity spectrum of the yield curve. Approximately 50 percent of the outstanding debt matures within the next three years while 21 percent falls within the maturity range 20-35 years (see Figure A1).

For this study five key rates were considered for varying reasons, the 1 year and 5 year were chosen as the major share of the government's domestic bond portfolio was at the short end, the 10 year key rate was reasonably viewed as a point along the curve ideal for conducting various shifts in the shape of the curve. For example the butterfly shift of the curve as well as a tilt of the curve could be facilitated by fixing the ten year key rate. The 20 and 30 year key rates provides useful analysis of the long end of the curve and are in line with the long-term maturity's share of the government's fixed income portfolio.

The result of our key rate application is presented in Figure (3). To calculate the KRD for the bond portfolio a shift of 100 basis points was applied to each of the key rates. Then, for each key a weight was assigned to each maturity conditional on the portfolio maturity spectrum. So for example, rates that had time to maturity of 1 year or less were assigned a weight that represents the share of nominal issues that mature within 1 year. Likewise, rates 1 to 2 years was assigned a weight of nominal issues that mature one to two years.

Figure 3: Key Rate Duration



As evident in Figure (3), the portfolio has a larger expositions over the medium to long-term. Specifically, the exposition for the 30-year key rate dominates the bond portfolio followed by the 20-year key rate.[9] This means that the bond portfolio is more sensitive to changes in the long end of the yield curve. Zeballos (2011) pointed out in a recent study that a concentration in the KRD at long end of the term structure may indicate an expectation of the flattening of the yield curve.[10]

6.2 Stress Testing Application of Yield Curve Shifts

As part of the Bank's interest rate stress test, scenario shifts in the yield curve are considered. This paper utilizes key rates to conduct parallel and non-parallel shifts in the yield curve. For a parallel shift in the yield curve, equal shifts in the selected key rates are considered. Nonparallel shifts in the yield curve amount to unequal shifts in the key rates. Specifically, an upward tilt of the yield curve at the 10-year key rate is achievable through an upward shift in key rates to the left of the 10-year key rate while simultaneously shifting the key rates to the right downwards. In

[9] A KRD of 50 for the 30-year key rate means that a 100 basis points change in the 30-year key rate would lead to 50 percent reduction in the weighted aggregated value of the GOJ domestic JMD portfolio cash flows that have a maturity period greater than 20 years.

[10] Similarly, the KRC for the bond portfolio was also calculated. The result of the KRC was in some sense similar to the outcome of the portfolio's KRD and are not included in the analysis for ease of explanation.

the case of the domestic fixed income sovereign issues, four cases are considered for illustration: (1) a parallel upward shift of the yield curve, (2) a flattening of the curve at the short end up to 10-year, (3) an increase in premiums for medium tenors and (4) a steepening of the curve at the long end of the maturity spectrum. The assessment of each scenarios will be conducted based on changes of stress levels of 20, 50 and 100 percent in the yields, respectively.

6.2.1 An Upward Parallel Shift of the Yield Curve

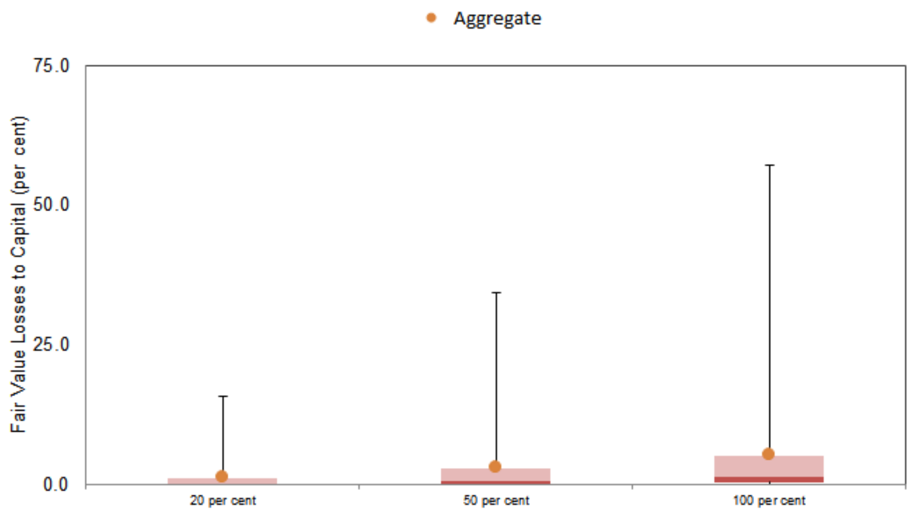
A parallel shift of the curve is supported by the notion of investors requiring equal premiums across the term structure due to higher perceived risk of government's ability to repay its debt. Such shift of the curve is accomplished by increasing the key rates by similar amount. The study considered 20, 50 and 100 percent increases in the key rates simultaneously across the estimated term structure. The new yield curve was then used to evaluate fair value losses[11] for portfolio holding of deposit taking institutions (DTIs), securities dealers and insurance companies.[12] The results of the parallel shift of the curve showed an impairment to capital base of DTIs of 5.4 percent resulting from a 100 shock to the yield curve (see Figure 4).[13]

[11]Fair value loss is define as the difference in value of GOJ domestic J$ portfolio holdings resulting from changes in yields.

[12]Currently, the deposit taking subsector comprises of six commercial banks, three building societies and two merchant banks. These institutions account for approximately 50 percent of the total financial system's assets.

[13]Impairment to capital for each sub-sector is define as the fair value loss divided by total accounting capital holding.

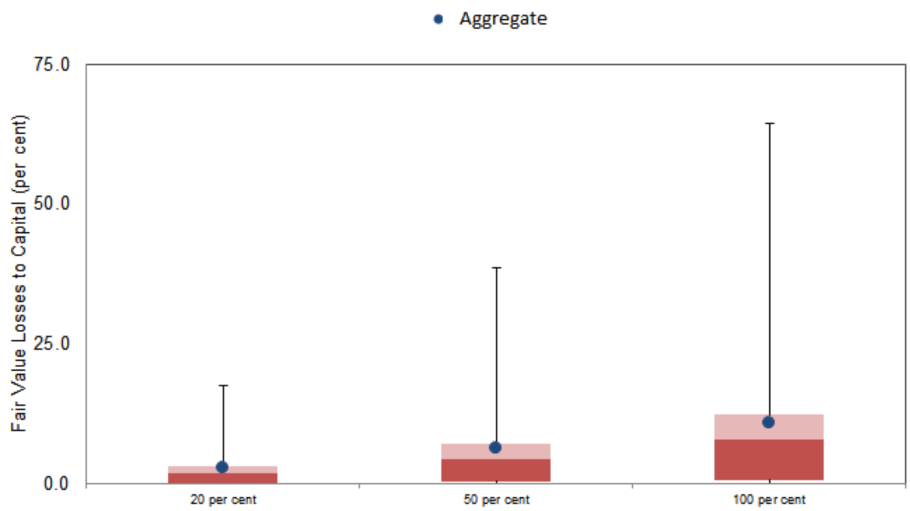
Figure 4: Box-plot of the Ratio of Fair Value losses to Capital for the Deposit Taking Institution Sector for Parallel Shifts of the Yield Cuve



A 20 percent increase in the term structure had a marginal impact on the fair value losses of the DTI sector (1.4 percent loss in capital) while at a 50 percent shock levels, impairments to capital was 3.1 percent (see Table B1 in Appendix B). The impact of the 100 percent shock threshold level on individual institutions within the DTI sector resulted in no significant impairment to their capital adequacy ratio (CAR); hence, indicating that the DTI sector is adequately capitalized to withstand such shocks in the yields on government's domestic issues.

The result of the analysis revealed that securities dealers were more susceptible to parallel shifts of the curve than DTIs. The sector's impairment to capital from a 100 percent upward shift of the term structure was 19.4 percent (see Table B1 in Appendix B). A 20 percent increase in the term structure would resulted in an impairment to securities dealers capital of 5.1 percent (see Figure 5) while a 50 percent increase in rates resulted in impairment of 11.3 percent.

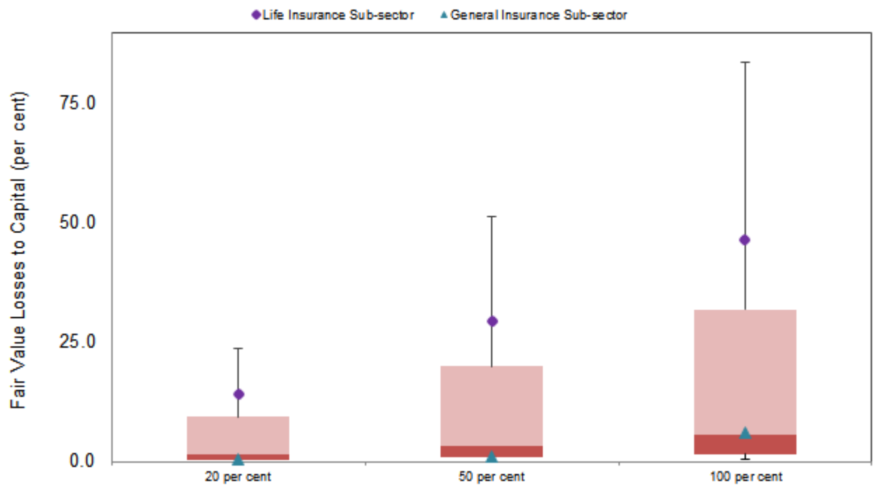
Figure 5: Box-plot of the Ratio of Fair Value losses to Capital for the Securities Dealer Sector for Parallel Shifts of the Yield Cuve



At the 50 percent shock level, one institution fell below the CAR prudential minimum level of 10 percent. The outcome further deteriorated at the 100 shock level where two institutions fell below the CAR prudential minimum level.

An assessment of the insurance industry revealed that fair value losses from a 100 increase in rates across the term structure accounted for 46.7 per cent of the life insurance sub-sector capital base. Exposure to the life insurance subsector on the other hand was less than 10 per cent of their capital base (see Table B1 in Appendix B). At the 100 per cent shock level, fair value losses across all three sectors of the market was highest for the insurance sector (specifically the life insurance sector, which accounted for 54 percent of total losses of JMD 47.6 million).

Figure 6: Box-plot of the Ratio of Fair Value losses to Capital for the Insurance Sector for Parallel Shifts of the Yield Cuve



6.2.2 Flattening of the Yield Curve at the Short End

A hypothetical flattening of the yield curve were considered, in which the 1-year and 5-year key rates increase by 20, 50 and 100 percent, respectively. Such movement in the curve would result in greater impact on portfolios holdings that are concentrated within maturities of up to 5 years. Exposure to the flattening of the curve at the short end was minimal for DTIs and securities dealer sectors. At the 100 per cent shock level fair value losses for DTIs and securities dealers amounted to 2.5 percent and 3.7 percent of their capital base, respectively (see Figures 7 and 8 and Table B1 in Appendix B).Similarly, exposure for the insurance sector was also minimal in comparison to a parallel shift of the curve (see Figure 9). Across the insurance sub-sectors, exposures at the respective shock levels were higher for the life insurance sub-sector. Additionally, across the market, the life insurance sector had the greatest exposure to the stress testing of the short-end of the curve followed by the DTI sector.

Figure 7: Box-plot of the Ratio of Fair Value losses to Capital for the Deposit Taking Institution Sector for Non-parallel Shifts of the Yield Cuve

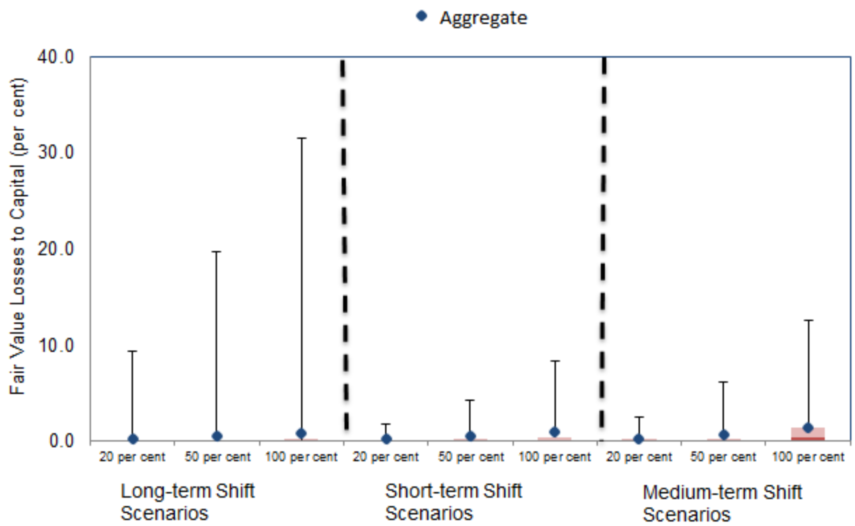
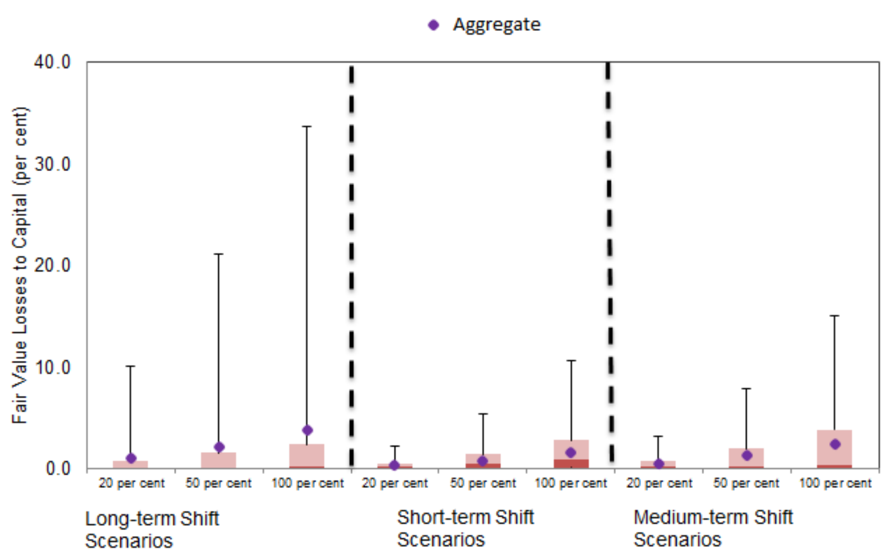


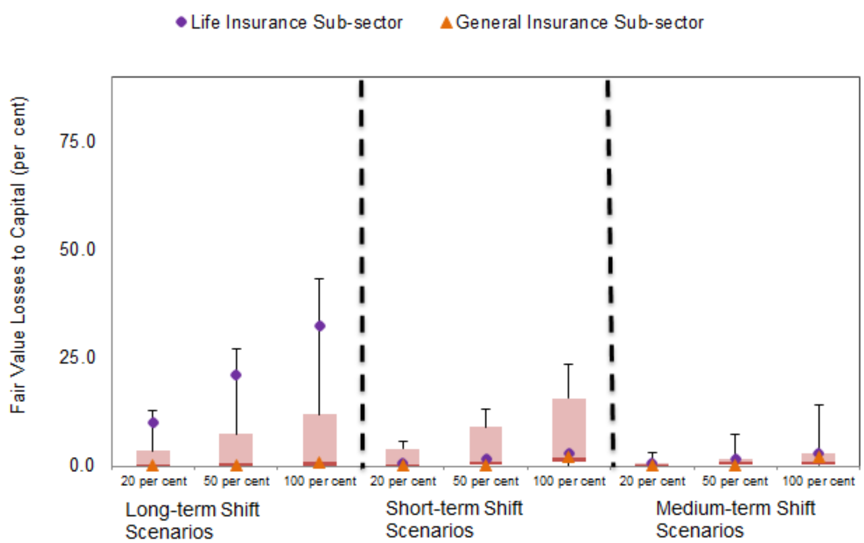
Figure 8: Box-plot of the Ratio of Fair Value losses to Capital for the Securities Dealers Sector for Non-parallel Shifts of the Yield Cuve



6.2.3 An increase in Premiums for medium tenures along the curve

A hypothetical increase in yields along the medium term tenures (i.e. 5 to 10 years) of the curve were considered as an increase in the demand for premiums along these tenors by investors. To simulate such changes in the yield curve the 5-year key rate was adjusted upwards at the respective shock levels. The adjustment in the five year key rate would impact yields that are greater than the 1-year key rate up to the 5-year key rate and above the 5-year key rate but less than the 10-year key rate. The fair value exposure to the movement along the curve was similar to that of a flattening of the curve at the short end for the insurance sector (see Figure 9 and Table B1 in Appendix B). While for the DTIs and securities dealers, such movement along the curve would resulted in lower exposure when compared to a flattening of the curve at the short end. At the 100 percent shock level, fair value losses relative to capital were 1.5 and 2.4 percent for DTIs and securities dealers, respectively (see Figures 7 and 8). Similarly, exposure to movements in the medium term tenures was greatest for the life insurance sub-sector across the market.

Figure 9: Box-plot of the Ratio of Fair Value losses to Capital for the Insurance Sector for Non-parallel Shifts of the Yield Cuve



6.2.4 A steepening of the curve at the long end of the maturity spectrum

A hypothetical increase in yields along the long end (i.e. above 10 years) of the curve were considered reflecting increase uncertainty of long-term macroeconomic conditions by investors. To simulate such movements in the yield curve, the 20-year and 30-year key rates were stressed at the respective shock levels. Relative to prior segmented shifts along the curve, exposures for the life insurance sub-sector was largest for shifts at the long end of the yield curve. At the 100 per cent shock level, fair value losses from such movement along the curve was 32.2 percent of capital for the and life insurance sub-sector (see Figure 8 and 9 and Table B1 in Appendix B). Conversely, relative to prior segmented shifts along the yield curve, exposures for DTIs and general insurance sub-sector were smallest for shift at the long end of the maturity spectrum. At the 100 percent stress level, fair value losses relative to capital were 0.9 percent for DTIs which was the same result for the general insurance sub-sector (see Figures 7 and 9).

From the respected shifts of the yield curve, it was observed that a parallel shift of the curve would have the largest impact of the fair value of the portfolio holdings of GOJ domestic securities across the respective sectors in the above analysis. In relation to non-parallel shifts of the yield curve, the results of the analysis were to some extent consistent with the fundamental market practice of the respective sectoral market participants. The life insurance sub-sector was more vulnerable to the long end of the maturity spectrum which is reflective of the appetite of their investment horizon. The DTIs and securities dealers on the other hand were more vulnerable to the medium term segment of the yield curve while the general insurance sub-sector vulnerability was equally weighted across the short term and medium term segments of the yield curve.

7 Conclusion

This paper estimated the GOJ domestic yield curves from 2014 to 2016 at quarterly frequency. The estimation of the curves was based on the Svensson model. The model fits the GOJ bond price data well without being over-parameterized and thus provides a consistent picture of GOJ's domestic yield curve evolution. The results from the estimation of the GOJ zero-coupon spot rate curve shows upward sloping yield curve. With the exception of 2014Q4, investors' preferences along the curve varies inversely across the 1 to 10 years and 10 to 30 years segments maturity spectrum of the GOJ domestic JMD debt portfolio.

Additionally, the estimated yield curve was utilized in an interest rate risk analysis for selected financial market participant sectors in Jamaica. As a risk assessment exercise, the study investigated the impact of parallel and non-parallel shifts of the yield curve on the portfolio holdings of selected domestic financial market participant sectors. The approach of the study relies on the KRD model for interest rate risk management. The choice of the KRD model was motivated by non-parallel shift scenarios for the yield curve.

The results from a parallel shift of the estimated yield curve showed that the life insurance sub-sector was more exposed to such movements in GOJ domestic bond yields relative to other market participant groups. In relation to non-parallel shifts of the curve, DTIs and securities dealers were more vulnerable to shifts in medium term segment of the yield curve. The life insurance sub-sector was more exposed to the long end of the yield curve while the general insurance sub-sector exposures were equally weighted across the short to medium term segment of the curve. The results of the assessment provides useful insights on the financial market structure, which was consistent with market expectation on the investment horizon for these participants.

The key rate model is a very useful tool for hedging interest rate risk and is used by market participants along with other tools. In light of the model's application, there are limitations to its use. Firstly, the choice of key rates is somewhat subjective. Thus the model offers no guidance on the choice of the risk factor to be used despite its importance. As a circumvention to this shortcoming of the model, different numbers and choices of key rates may be selected conditional on the maturity structure of the portfolio under consideration.
Secondly, the shift in the individual key rates provides an implausible yield curve shape. Further the shift in the key rates assumes strong correlation of the neighboring rates which may not always be the case. In addressing this short coming of the model, Johnson and Meyer (1989) proposed the *Partial Derivative Approach (PDA)*. The PDA assumes that the forward rate curve is split up into many linear segments and all forward rates within each segment are assumed to change in a parallel way. Under the PDA each forward rate affects the present value of all the cash flows occurring within or after the term of the forward rate.

Lastly, the key rate model does not take into account past movements in past yield curves hence making the model inefficient in describing the dynamics of term structure because historical volatilities of interest rates provide useful information.

Appendix A

Figure A10: Holdings of GOJ Domestic JMD Issue by Deposit Taking Institutions and Securities Dealers for the period March 2014 to March 2016

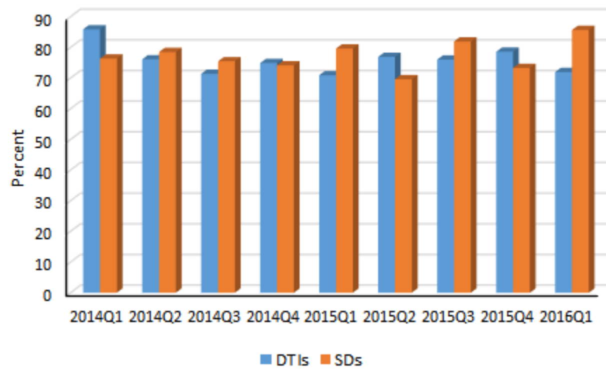


Figure A11: Disaggregation of the share of GOJ Domestic JMD Issue by Maturity as at March 2016

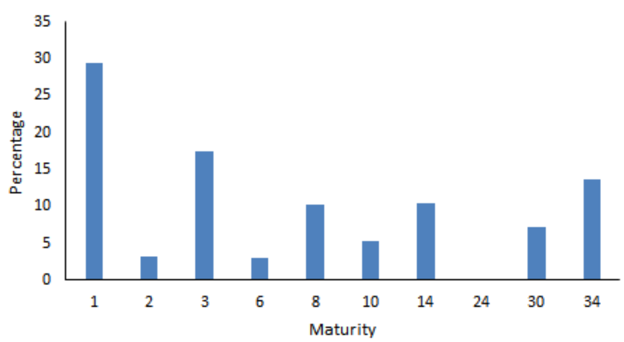


Table A2: Parameter Output

Estimated Parameters for the period 2014Q1 : 2016Q1 (Actual Values)						
Date	β_0	β_1	β_2	β_3	λ	γ
2014Q1	0.17	-0.13	-0.15	0.16	0.41	3.81
2014Q2	0.15	-0.11	-0.19	0.16	0.64	2.40
2014Q3	0.08	-0.00	-18.86	18.96	0.18	0.18
2014Q4	0.19	-0.13	28.22	-28.40	0.59	0.59
2015Q1	0.20	-0.12	-0.12	-0.33	2.52	0.38
2015Q2	0.17	-0.11	-22.06	21.93	0.74	0.74
2015Q3	0.15	-0.08	-0.09	-0.19	3.71	0.60
2015Q4	0.18	-0.09	-0.15	-0.09	0.34	7.56
2016Q1	0.15	-0.11	-19.53	19.42	0.83	0.83

Appendix B

Table B3: Fair value losses relative to capital from key rate shifts of the estimated yield curve as March 2016

		Shock Levels (Percent)		
		20	50	100
Parallel Upward Shift of the Curve	DTIs	1.4	3.1	5.4
	SDs	5.1	11.3	19.4
	LIs	14.2	29.6	46.7
	GIs	0.5	1.2	6.2
Flattening of the curve at the short-end	DTIs	0.6	1.4	2.8
	SDs	0.9	2.3	4.3
	LIs	1.4	3.3	6.4
	GIs	0.3	0.6	3.5
Increase in medium term tenures along the curve	DTIs	0.8	1.8	3.1
	SDs	1.5	3.4	6.1
	LIs	3.5	7.9	13.7
	GIs	0.3	0.6	3.4
Steepening of the curve at the long-end	DTIs	0.3	0.6	0.9
	SDs	1.0	2.3	3.8
	LIs	10.2	21.0	32.3
	GIs	0.1	0.2	0.9

Key: DTIs- Deposit Taking Institutions sector, SDs Securities Dealer sector, LIs- Life Insurance sub-sector, GIs - General Insurance sub-sector.

References

[1] Bomfin A. N., 2003. Monetary Policy and the Yield Curve.*Federal Reserve Board, Washington, D.C.*

[2] BIS, 2005. Zero-coupon yield curves: technical documentation *Bank for International Settlements, BIS Papers No 25.*

[3] Campbell J. and Shiller R., 1991. Yield spreads and interest rate movements: a bird's eye view, *Review of Economic Studies, 58, pp 495-514.*

[4] Coleman T. F. and Li Y., 1996. An interior trust region approach for nonlinear minimization subject to bounds, *Society for Industrial and Applied Mathematics (SIAM) Journal on optimization* Vol. 6.

[5] Cairns A. and Pritchard D., 2001. Stability of Models for the Term Structure of Interest Rates with Application to German Market Data, *British Actuarial Journal.*

[6] Estrella A. and Trubin M. R., 2006. The Yield Curve as a Leading Indicator: Some Practical Issues.*Federal Reserve Bank of New York.*

[7] Fisher M., 1996. Fitting and interpreting the US yield curve at the Federal Reserve Board.

[8] Gimeno R. and Nave J. M., 2009. A genetic algorithm estimation of the term structure of interest rates. *Computational Statistics and Data Analysis, 53:2236 - 2250.*

[9] Gurkaynak R. S. and Sack B. and Wright J. H., 2007. The U.S. Treasury Yield Curve: 1961 to the Present. *Federal Reserve Board, Washington, D.C.*

[10] Ho T., 1992. Key Rate Durations: Measures of Interest Rate Risks. *Journal of Fixed Income*, September 1992, p. 29-44.

[11] Kladivko K., 2010. The Czech Treasury Yield Curve from 1999 to the Present. *Czech Journal of Economics and Finance, 60, 2010, no. 4*

[12] Langrin R. B., 2007. State-Space Estimation of Multi-Factor Models of the Term Structure: An Application to Government of Jamaica Bonds. *Bank of Jamaica*

[13] Macaulay F. 1938. The Movements of Interest Rates. Bond Yields and Stock Prices in the United States since 1856, *New York: National Bureau of Economic Research.*

[14] Nawalkha S. K. and Soto G. M. and Beliaeva N. A., 2005. Interest Rate Risk Modeling: The Fixed Income Valuation Course, *Wiley & Sons, Inc., Hoboken, New Jersey.*

[15] Nelson C. R. and Siegel A. F., 1987. "Parsimonious Modeling of Yield Curves", *The Journal of Business*, 1987, vol. 60, no.4

[16] Svensson L. E., 1994. Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994, Centre for Economic Policy Research, Discussion Paper No 1051.

[17] Tracey M. Principal Component Value at Risk: An application to the measurement of the interest rate risk exposure of Jamaican Banks to Government of Jamaica (GOJ) Bonds. *Bank of Jamaica*

[18] Waggoner, D., 1994. Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices. Federal Reserve Bank of Atlanta Working Paper, no. 97-10

[19] Zeballos D., 2011. Market Risk Measurement: Key Rate Duration as an asset allocation instrument. *Central Bank of Bolivia*